Unit 3, Lesson 9

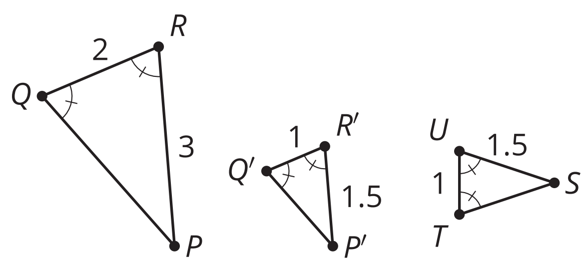
Conditions for Triangle Similarity

Let's prove that some triangles are similar.

9.1 Math Talk: Angle-Side-Angle as a Helpful Tool

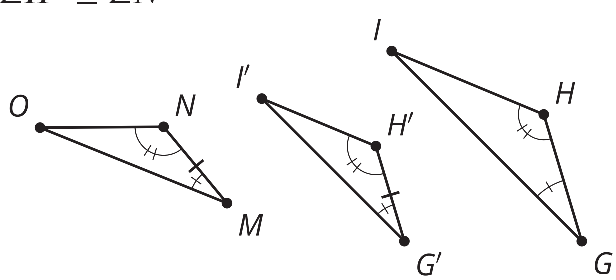
Justify each statement.

$\angle Q \cong \angle R \cong \angle Q' \cong \angle R' \cong \angle T \cong \angle U$



- Triangle $P'Q'R'$ is congruent to triangle STU.
- Triangle PQR is similar to triangle STU.

$\angle G \cong \angle G' \cong \angle M, \angle H \cong \angle H' \cong \angle N$



- Triangle $G'H'I'$ is congruent to triangle MNO.
- Triangle GHI is similar to triangle MNO.

9.2 How Many Pieces?

For each problem, draw 2 triangles that have the listed properties. Try to make them as different as possible.

1. One angle is 45 degrees.

2. One angle is 45 degrees, and another angle is 30 degrees.

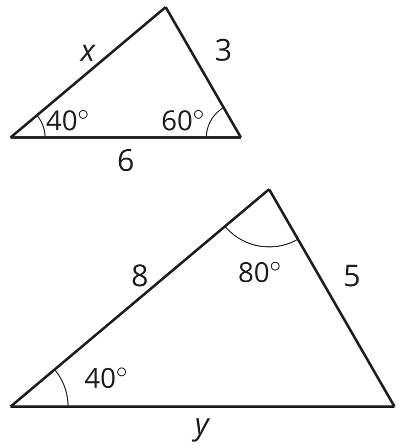
3. One angle is 45 degrees, and another angle is 30 degrees. The lengths of a pair of corresponding sides are 2 cm and 6 cm.

4. Compare your triangles with the other triangles from your group. Do any of the conditions guarantee that the triangles will be similar? Make a conjecture.

5. Prove your conjecture.

9.3 Any Two Angles?

Here are 2 triangles. One triangle has a 60 degree angle and a 40 degree angle. The other triangle has a 40 degree angle and an 80 degree angle.



1. How can you show that the triangles are similar?

2. How long are the sides labeled x and y?

Are you ready for more?

Under what conditions is there an Angle-Angle Quadrilateral Similarity Theorem? What about an Angle-Angle-Angle Quadrilateral Similarity Theorem? Explain or show your reasoning.

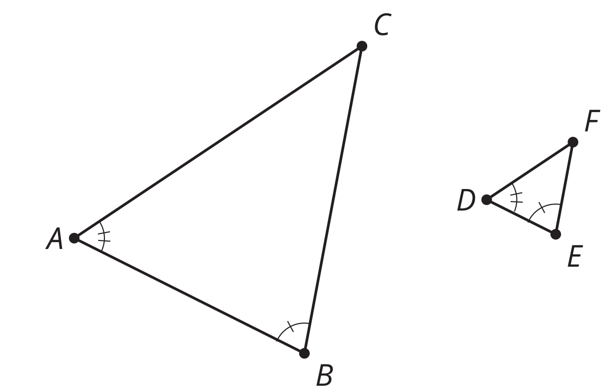
Lesson 9 Summary

We already know that when two figures are congruent there is a sequence of rigid motions that takes one figure onto the other. So, if a dilation takes Figure A to an image that is congruent to Figure B, then Figure A and Figure B are similar because there is a sequence of a dilation and rigid motions that takes Figure A onto Figure B.

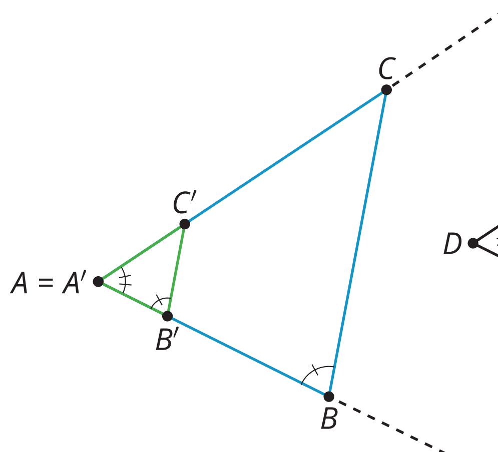
We can use this idea to show that when two angles of one triangle are congruent to two angles of a second triangle, then the two triangles are similar. We call this the Angle-Angle Triangle Similarity Theorem.

In the diagram, angle A is congruent to angle D, and angle B is congruent to angle E. If a sequence of rigid motions and dilations moves the first figure so that it fits exactly over the second, then we have shown that the Angle-Angle Triangle Similarity Theorem is true.

$\angle A \cong \angle D, \angle B \cong \angle E$



Dilate triangle ABC by the ratio $\frac{DE}{AB}$, so that $A'B'$ is congruent to DE. Now triangle $A'B'C'$ is congruent to triangle DEF by the Angle-Side-Angle Triangle Congruence Theorem, which means that there is a sequence of rotations, reflections, and translations that takes $A'B'C'$ onto DEF.



Therefore, a dilation followed by a sequence of rotations, reflections, and translations will take triangle ABC onto triangle DEF, which is the definition of similarity. We have shown that a dilation and a sequence of rigid motions takes triangle ABC to triangle DEF, so the triangles are similar.