



# More Arithmetic with Complex Numbers

Let's practice adding, subtracting, and multiplying complex numbers.

## 11.1 Which Three Go Together: Complex Expressions

Which three go together? Why do they go together?

A

$$2i \cdot i$$

B

$$(1 + i) + (1 - i)$$

C

$$(1 + i)^2$$

D

$$(1 + 2i)(1 - 2i)$$

## 11.2 Powers of $i$

1. Write each power of  $i$  in the form  $a + bi$ , where  $a$  and  $b$  are real numbers. If  $a$  or  $b$  is zero, you do not need to write that part of the number. For example,  $0 + 3i$  can be expressed as  $3i$ .

$$i^0$$

$$i^1$$

$$i^2$$

$$i^3$$

$$i^4$$

$$i^5$$

$$i^6$$

$$i^7$$

$$i^8$$



2. Use any patterns you noticed to rewrite  $i^{100}$  in a similar way. Explain your reasoning.

3. Use any patterns you noticed to rewrite  $i^{38}$  in a similar way. Explain your reasoning.



### Are you ready for more?

1. Write each power of  $1 + i$  in the form  $a + bi$ , where  $a$  and  $b$  are real numbers. If  $a$  or  $b$  is zero, you do not need to write that part of the number. For example,  $0 + 3i$  can be expressed as  $3i$ .

$$(1 + i)^0$$

$$(1 + i)^1$$

$$(1 + i)^2$$

$$(1 + i)^3$$

$$(1 + i)^4$$

$$(1 + i)^5$$

$$(1 + i)^6$$

$$(1 + i)^7$$

$$(1 + i)^8$$

2. Compare and contrast the powers of  $1 + i$  with the powers of  $i$  when plotted on the complex plane. What is the same? What is different?

## 11.3

## Add 'Em Up (or Subtract or Multiply)

For each row, your partner and you will each rewrite an expression so it has the form  $a + bi$ , where  $a$  and  $b$  are real numbers. You and your partner should get the same answer. If you disagree, work to reach an agreement.

partner A	partner B
$(7 + 9i) + (3 - 4i)$	$5i(1 - 2i)$
$2i(3 + 4i)$	$(1 + 2i) - (9 - 4i)$
$(4 - 3i)(4 + 3i)$	$(5 + i) + (20 - i)$
$(2i)^4$	$(3 + i\sqrt{7})(3 - i\sqrt{7})$
$(1 + i\sqrt{5}) - (-7 - i\sqrt{5})$	$(-2i)(-\sqrt{5} + 4i)$
$(\frac{1}{2}i)(\frac{1}{3}i)(\frac{3}{4}i)$	$(\frac{1}{2}i)^3$

## Lesson 11 Summary

Suppose we want to write the product  $(3 + 5i)(7 - 2i)$  in the form  $a + bi$ , where  $a$  and  $b$  are real numbers. For example, we might want to compare our solution with a partner's, and having answers in the same form makes that easier. Using the distributive property,

$$\begin{aligned}(3 + 5i)(7 - 2i) &= 21 - 6i + 35i - 10i^2 \\ &= 21 + 29i - 10(-1) \\ &= 21 + 29i + 10 \\ &= 31 + 29i\end{aligned}$$

Keeping track of the negative signs is especially important since it is easy to mix up the fact that  $i^2 = -1$  with the fact that  $-i^2 = -(-1) = 1$ .

Next, suppose we want to write the difference  $(-6 + 3i) - (2 - 4i)$  as a single complex number in the form  $a + bi$ . Distributing the negative and combining like terms, we get:

$$\begin{aligned}(-6 + 3i) - (2 - 4i) &= -6 + 3i - 2 - (-4i) \\ &= -8 + 3i + 4i \\ &= -8 + 7i\end{aligned}$$

Again, it is important to be precise with negative signs. It is a common mistake to subtract  $4i$  rather than subtracting  $-4i$ .