Completing the Square and Complex Solutions

Let's find complex solutions to quadratic equations by completing the square.

13.1 Creating Quadratic Equations

Match each equation in standard form to its factored form and its complex solutions.

1. $x^2 - 25 = 0$
2. $x^2 - 5 = 0$
3. $x^2 + 25 = 0$

- $(x - 5i)(x + 5i) = 0$
- $(x - 5)(x + 5) = 0$
- $(x - \sqrt{5})(x + \sqrt{5}) = 0$

- $\sqrt{5}$, -$\sqrt{5}$
- 5, -5
- $5i$, -$5i$

13.2 Sometimes the Solutions Aren't Real Numbers

What are the complex solutions to these equations? Check your solutions by substituting them into the original equation.

1. $(x - 5)^2 = 0$
2. $(x - 5)^2 = 1$
3. $(x - 5)^2 = -1$

13.3 Finding Complex Solutions

Solve these equations by completing the square to find all complex solutions.

1. $x^2 - 8x + 13 = 0$

2. $x^2 - 8x + 19 = 0$

Are you ready for more?

For which values of a does the equation $x^2 - 8x + a = 0$ have two real solutions? One real solution? No real solutions? Explain your reasoning.

13.4 Can You See the Solutions on a Graph?

1. How many real solutions does each equation have? How many non-real solutions?
   a. $x^2 - 8x + 13 = 0$

   b. $x^2 - 8x + 16 = 0$

   c. $x^2 - 8x + 19 = 0$

2. How do the graphs of these functions help us answer the previous question?
   a. $f(x) = x^2 - 8x + 13$

   b. $g(x) = x^2 - 8x + 16$

   c. $h(x) = x^2 - 8x + 19$

Lesson 13 Summary

Sometimes quadratic equations have real solutions, and sometimes they do not. Here is a quadratic equation with x^2 equal to a negative number (assume k is positive):

$$x^2 = \text{-}k$$

This equation has imaginary solutions $i\sqrt{k}$ and $\text{-}i\sqrt{k}$. By similar reasoning, an equation of the form:

$$(x-h)^2 = \text{-}k$$

has non-real solutions if k is positive. In this case, the solutions are $h + i\sqrt{k}$ and $h - i\sqrt{k}$.

It isn't always clear just by looking at a quadratic equation whether the solutions will be real or not. For example, look at this quadratic equation:

$$x^2 - 12x + 41 = 0$$

We can always complete the square to find out what the solutions will be:

$$\begin{aligned} x^2 - 12x + 36 + 5 &= 0 \\ (x-6)^2 + 5 &= 0 \\ (x-6)^2 &= \text{-}5 \\ x - 6 &= \pm i\sqrt{5} \\ x &= 6 \pm i\sqrt{5} \end{aligned}$$

This equation has non-real, complex solutions $6 + i\sqrt{5}$ and $6 - i\sqrt{5}$.

