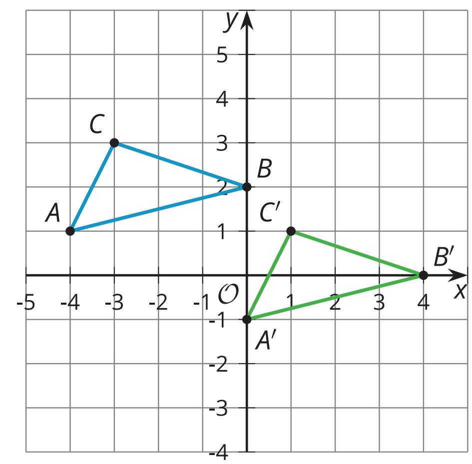
Unit 6 Family Support Materials

Coordinate Geometry

In this unit, your student will make connections between geometry and algebra by working in the coordinate plane with geometric concepts from prior units. The coordinate grid imposes a structure that can provide new insights into ideas students have previously explored.

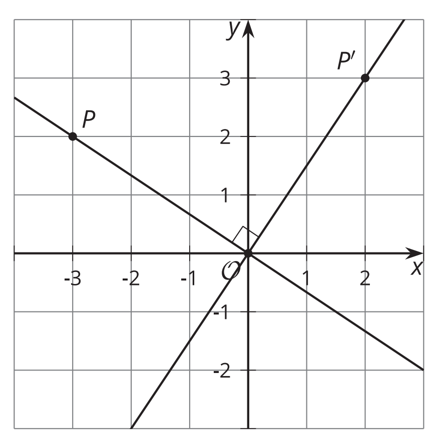
Your student has already worked with transformations. Here, they'll think about transformations as functions that take points in the plane as inputs and give other points as outputs. For example, the notation $(x, y) \rightarrow (x + 4, y - 2)$ means that to find the image for each point in a figure, we add 4 units to the x-coordinate and subtract 2 units from the y-coordinate. Let's apply this transformation to triangle ABC.

(x, y)	$(x + 4, y - 2)$
A : (-4, 1)	A' : (0, -1)
B : (0, 2)	B' : (4, 0)
C : (-3, 3)	C' : (1, 1)



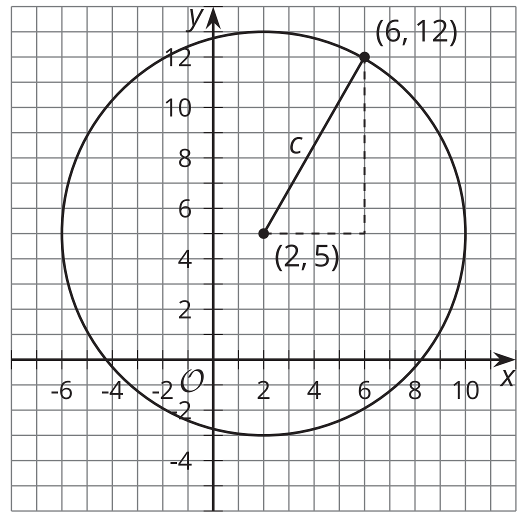
This transformation was a translation by the directed line segment from $(-4, 1)$ to $(0, -1)$, or informally, a translation 4 units right and 2 units down.

Transformations can also be used to analyze slopes of parallel and perpendicular lines. Suppose we draw a line passing through the point $P = (-3, 2)$ and the point $(0, 0)$, then apply the transformation $(x, y) \rightarrow (y, -x)$ to the line.



This rule rotates the line 90 degrees clockwise using the point $(0, 0)$ as a center. The center of rotation doesn't move, so $(0, 0)$ maps to itself. The image of point P is $P' = (2, 3)$. The slope of the original line is $-\frac{2}{3}$, and the slope of the image is $\frac{3}{2}$. The slopes are *opposite reciprocals* of one another. Your student will use this to prove that *any* two perpendicular lines that aren't horizontal and vertical have slopes that are opposite reciprocals.

The Pythagorean Theorem proves useful in the coordinate plane as well. Consider the circle with a center at $(2, 5)$ and a radius of 8 units. The point $(6, 12)$ appears to be on the circle. We can test if it really is on the circle by calculating the distance between this point and the center. Start by drawing a right triangle whose hypotenuse is the distance between the 2 points.



The lengths of the triangle's legs can be calculated by subtracting the coordinates of the points: The vertical leg is 7 units long, and the horizontal leg is 4 units long. Substitute these into the Pythagorean Theorem.

$a^2 + b^2 = c^2$

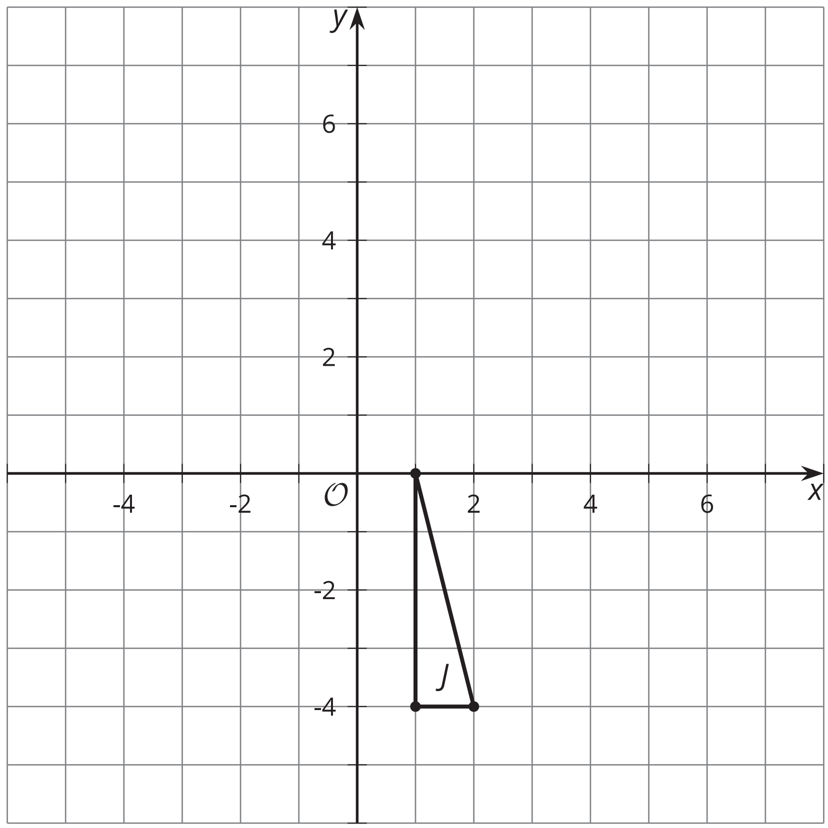
$4^2 + 7^2 = c^2$

$65 = c^2$

The distance between the points is the positive number that squares to make 65, or about 8.1 units. So, because it's not exactly 8 units away from the circle's center, the point $(6, 12)$ isn't actually on the circle.

Here is a task to try with your student:

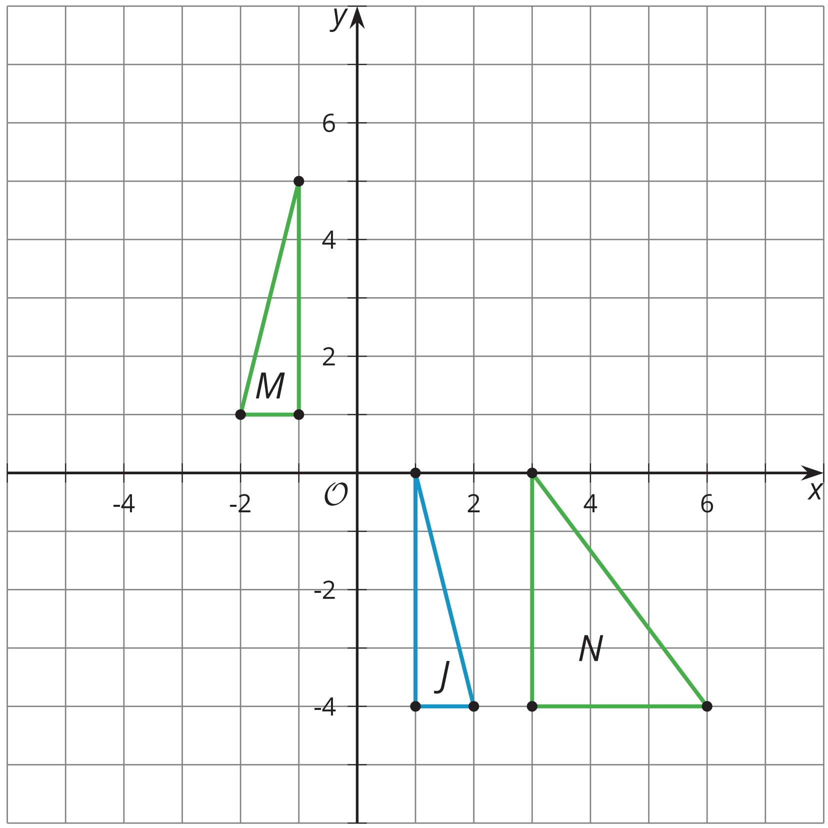
The image shows triangle J.



Apply each transformation rule to triangle J. Then, describe the transformation, and decide whether it produced a congruent image, a similar image, or neither.

1. Label the result of this transformation $M: (x, y) \rightarrow (-x, y + 5)$
2. Label the result of this transformation $N: (x, y) \rightarrow (3x, y)$

Solution:



1. This transformation was a reflection across the y-axis, then a translation by the directed line segment from $(-1, 0)$ to $(-1, 5)$. All 3 corresponding pairs of sides of the original and image triangles are congruent, so the 2 triangles are congruent (and therefore also similar) by the Side-Side-Side Triangle Congruence Theorem. This makes sense because reflections and translations are rigid motions.
2. This transformation was a horizontal stretch away from the y-axis by a factor of 3. The corresponding vertical sides of triangle J and triangle N are congruent, but the horizontal side of triangle N is 3 times as long as the corresponding side in triangle J. Since pairs of corresponding sides are neither all congruent nor all proportional, the 2 triangles are neither congruent nor similar.