Using Factors and Zeros

Let's write some polynomials.

7.1 More Than Factors

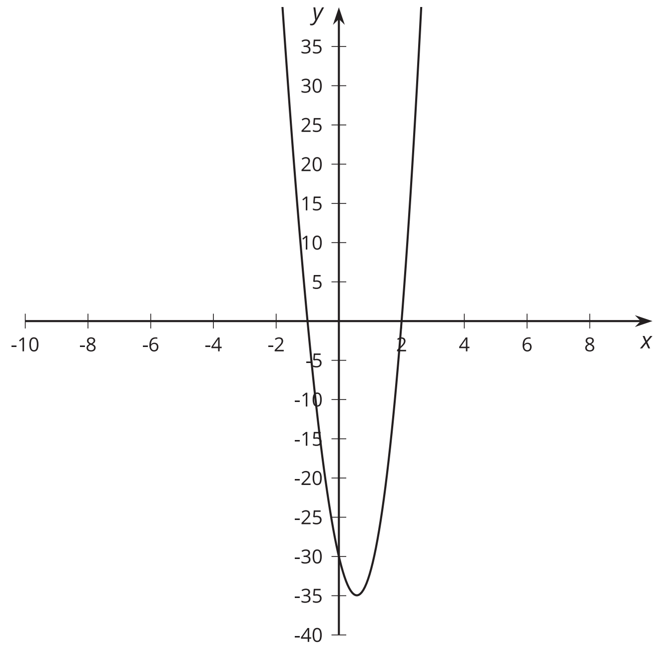
M and K are both polynomial functions of x, where
$M(x) = (x + 3)(2x - 5)$ and $K(x) = 3(x + 3)(2x - 5)$.

1. How are the two functions alike? How are they different?

2. If a graphing window of $-5 \leq x \leq 5$ and $-20 \leq y \leq 20$ shows all intercepts of a graph of $y = M(x)$, what graphing window would show all intercepts of $y = K(x)$?

7.2 Choosing Windows

Mai graphs the function p given by $p(x) = (x+1)(x-2)(x+15)$ and sees this graph.



She says, "This graph looks like a parabola, so it must be a quadratic."

1. Is Mai correct? Use graphing technology to check.

2. Explain how you could select a viewing window before graphing an expression like $p(x)$ that would show the main features of a graph.

3. Using your explanation, what viewing window would you choose for graphing $f(x) = (x+1)(x-1)(x-2)(x-28)$?

Are you ready for more?

Select some different windows for graphing the function $q(x) = 23(x - 53)(x - 18)(x + 111)$. What is challenging about graphing this function?

7.3 What's the Equation?

Write a possible equation for a polynomial whose graph has the following horizontal intercepts. Check your equation using graphing technology.

1. $(4, 0)$
2. $(0, 0)$ and $(4, 0)$
3. $(-2, 0)$, $(0, 0)$ and $(4, 0)$
4. $(-4, 0)$, $(0, 0)$, and $(2, 0)$
5. $(-5, 0)$, $\left(\frac{1}{2}, 0\right)$, and $(3, 0)$

Lesson 7 Summary

We can use the zeros of a polynomial function to figure out what an expression for the polynomial might be. One way to write a polynomial expression is as a product of linear factors.

For example, for a polynomial function Z that satisfies $Z(x) = 0$ when x is -1, 2, or 4, we could multiply together a factor that is 0 when $x = -1$, a factor that is 0 when $x = 2$, and a factor that is 0 when $x = 4$. It turns out that there are many possible expressions for $Z(x)$.

Using linear factors, one possibility is $Z(x) = (x + 1)(x - 2)(x - 4)$.

Another possibility is $Z(x) = 2(x + 1)(x - 2)(x - 4)$, since the 2 (or any other rational number) does not change what values of x make the function equal to 0.

We can test the three values -1, 2, and 4 to make sure that $Z(x)$ is 0 for those values. We can also graph both possible versions of Z and see that the graphs intercept the horizontal axis at -1, 2, and 4. Notice that while both functions have the same output at these three specific input values, they have different outputs for all other input values.