Solving Systems by Elimination (Part 2)

Let's think about why adding and subtracting equations work for solving systems of linear equations.

15.1 Is It Still True?

Here is a true equation: $50 + 1 = 51$.

1. Perform each of the following operations and answer these questions: What does each resulting equation look like? Is it still a true equation?
   a. Add 12 to each side of the equation.
   b. Add $10 + 2$ to the left side of the equation and 12 to the right side.
   c. Add the equation $4 + 3 = 7$ to the equation $50 + 1 = 51$.
2. Write a new equation that, when added to $50 + 1 = 51$, gives a sum that is also a true equation.
3. Write a new equation that, when added to $50 + 1 = 51$, gives a sum that is a false equation.

15.2 Classroom Supplies

A teacher purchased 20 calculators and 10 measuring tapes for her class and paid \$495. Later, she realized that she didn't order enough supplies. She placed another order of 8 of the same calculators and 1 more of the same measuring tape and paid \$178.50.

This system represents the constraints in this situation:

$$\begin{cases} 20c + 10m = 495 \\ 8c + m = 178.50 \end{cases}$$

1. Discuss with a partner:
   a. In this situation, what do the solutions to the first equation mean?
   b. What do the solutions to the second equation mean?
   c. For each equation, how many possible solutions are there? Explain how you know.
   d. In this situation, what does the solution to the system mean?
2. Find the solution to the system. Explain or show your reasoning.

3. To be reimbursed for the cost of the supplies, the teacher recorded: "Items purchased: 28 calculators and 11 measuring tapes. Amount: \$673.50."
   a. Write an equation to represent the relationship between the numbers of calculators and measuring tapes, the prices of those supplies, and the total amount spent.
   b. How is this equation related to the first two equations?

c. In this situation, what do the solutions of this equation mean?

d. How many possible solutions does this equation have? How many solutions make sense in this situation? Explain your reasoning.

15.3 A Bunch of Systems

Solve each system of equations without graphing and show your reasoning. Then, check your solutions.

A $\begin{cases} 2x + 3y = 7 \\ \text{-}2x + 4y = 14 \end{cases}$

B $\begin{cases} 2x + 3y = 7 \\ 3x - 3y = 3 \end{cases}$

C $\begin{cases} 2x + 3y = 5 \\ 2x + 4y = 9 \end{cases}$

D $\begin{cases} 2x + 3y = 16 \\ 6x - 5y = 20 \end{cases}$

Are you ready for more?

This system has three equations: $\begin{cases} 3x + 2y - z = 7 \\ \text{-}3x + y + 2z = \text{-}14 \\ 3x + y - z = 10 \end{cases}$

1. Add the first two equations to get a new equation.

2. Add the second two equations to get a new equation.

3. Solve the system of your two new equations.

4. What is the solution to the original system of equations?

Lesson 15 Summary

When solving a system with two equations, why is it acceptable to add the two equations, or to subtract one equation from the other?

Remember that an equation is a statement that says two things are equal. For example, the equation $a = b$ says a number a has the same value as another number b. The equation $10 + 2 = 12$ says that $10 + 2$ has the same value as 12.

If $a = b$ and $10 + 2 = 12$ are true statements, then adding $10 + 2$ to a and adding 12 to b means adding the same amount to each side of $a = b$. The result, $a + 10 + 2 = b + 12$, is also a true statement.

As long as we add an equal amount to each side of a true equation, the two sides of the resulting equation will remain equal.

We can reason the same way about adding variable equations in a system like this:

$$\begin{cases} d + f = 17 \\ \text{-}2d + f = \text{-}1 \end{cases}$$

In each equation, if (d, f) is a solution, the expression on the left of the equal sign and the number on the right are equal. Because $\text{-}2d + f$ is equal to -1:

- Adding $\text{-}2d + f$ to $d + f$ and adding -1 to 17 means adding an equal amount to each side of $d + f = 17$. The two sides of the new equation, $\text{-}d + 2f = 16$, stay equal.

  $$\begin{array}{rl} d + f = 17 & \\ \text{-}2d + f = \text{-}1 & + \\ \hline \text{-}d + 2f = 16 & \end{array}$$

  The d- and f-values that make the original equations true also make this equation true.

- Subtracting $\text{-}2d + f$ from $d + f$ and subtracting -1 from 17 means subtracting an equal amount from each side of $d + f = 17$. The two sides of the new equation, $3d = 18$, stay equal.

  $$\begin{array}{rl} d + f = 17 & \\ \text{-}2d + f = \text{-}1 & - \\ \hline 3d \quad\quad = 18 & \end{array}$$

  The f-variable is eliminated, but the d-value that makes both the original equations true also makes this equation true.

From $3d = 18$, we know that $d = 6$. Because 6 is also the d-value that makes the original equations true, we can substitute it into one of the equations and find the f-value.

The solution to the system is $d = 6, f = 11$, or the point $(6, 11)$ on the graphs representing the system. If we substitute 6 and 11 for d and f in any of the equations, we will find true equations. (Try it!)

