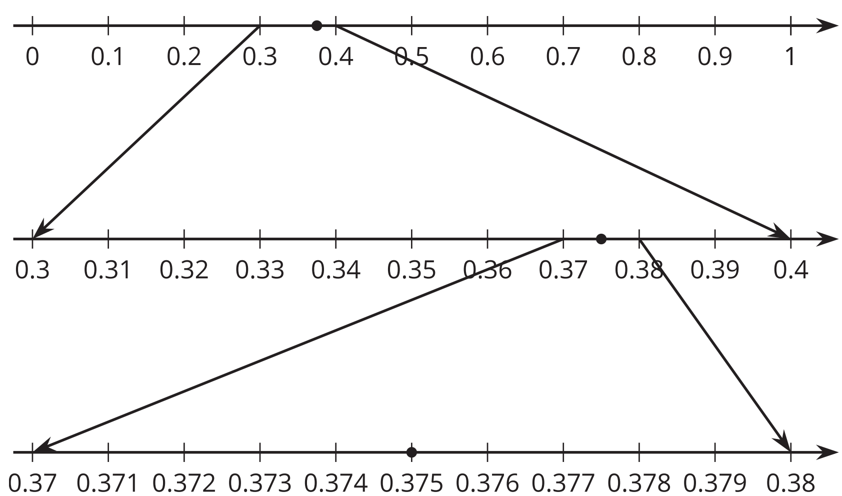
Decimal Representations of Rational Numbers

Let's learn more about how rational numbers can be represented.

16.1 Notice and Wonder: Number Lines

What do you notice? What do you wonder?



16.2 Rational Numbers as Fractions

Rational numbers can be written as positive or negative fractions. All of these numbers are rational numbers. Show that they are rational by writing them in the form $\frac{a}{b}$ or $-\frac{a}{b}$.

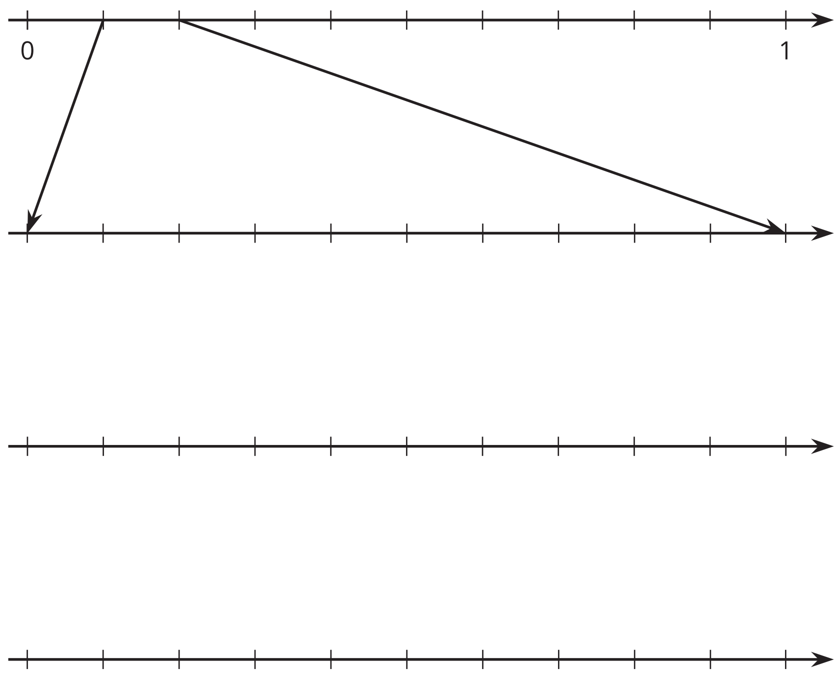
1. 0.2
2. $-\sqrt{4}$
3. 0.333
4. $\sqrt[3]{1,000}$
5. -1.000001
6. $\sqrt{\frac{1}{16}}$

16.3 Rational Numbers as Decimals

All rational numbers have decimal representations, too. Find the decimal representation of each of these rational numbers.

1. $\frac{7}{5}$
2. $\frac{999}{1,000}$
3. $\frac{111}{2}$
4. $\sqrt[3]{\frac{1}{8}}$
5. $\frac{3}{8}$

16.4 Zooming In on $\frac{2}{11}$



1. On the topmost number line, label the tick marks. Next, find the first decimal place of $\frac{2}{11}$ using long division and estimate where $\frac{2}{11}$ should be placed on the top number line.
2. Label the tick marks of the second number line. Find the next decimal place of $\frac{2}{11}$ by continuing the long division and estimate where $\frac{2}{11}$ should be placed on the second number line. Add arrows from the second to the third number line to zoom in on the location of $\frac{2}{11}$.
3. Label the tick marks of the remaining number lines. Continue using long division to calculate the next two decimal places, and plot them on the remaining number lines.
4. What do you think the decimal expansion of $\frac{2}{11}$ is?

Are you ready for more?

Let $x = \frac{25}{11} = 2.272727...$ and let $y = \frac{58}{33} = 1.75757575...$.

For each of the following questions, first decide whether the fraction or decimal representations of the numbers are more helpful to answer the question, and then find the answer.

- Which of x or y is closer to 2?
- Find x^2.

Lesson 16 Summary

We learned earlier that rational numbers can be written as a positive or negative fraction. For example, $\frac{3}{4}$ and $-\frac{5}{2}$ are both rational numbers. A complicated-looking numerical expression can also be a rational number as long as the value of the expression is a positive or negative fraction. For example, $\sqrt{64}$ and $-\sqrt[3]{\frac{1}{8}}$ are rational numbers because $\sqrt{64} = 8$ and $-\sqrt[3]{\frac{1}{8}} = -\frac{1}{2}$.

Rational numbers can also be written using decimal notation. Some have finite decimal expansions, like 0.75, -2.5, or -0.5. Other rational numbers have infinite decimal expansions, like 0.7434343 . . . , where the 43s repeat forever. This is called a **repeating decimal**. A repeating decimal has digits that keep going in the same pattern over and over, and these repeating digits are marked with a line above them. For example, we would write 0.7434343 . . . as $0.7\overline{43}$. The bar tells us which part repeats forever.

The decimal expansion of a number helps us plot it accurately on a number line divided into tenths. For example, $0.7\overline{43}$ should be between 0.7 and 0.8. Each additional decimal digit increases the accuracy of our plotting. So the number $0.7\overline{43}$ is between 0.743 and 0.744.

