



# El tamaño de un ángulo, en grados

## Standards

Building On	4.NBT.A.1
Addressing	4.MD.C.5.a, 4.MD.C.7
Building Toward	4.MD.C.5, 4.MD.C.6, 4.MD.C.7

## Instructional Routines

- What Do You Know about \_\_\_\_?

## Goals

- Comprehend (in spoken and written language) the meaning of the term “degree.”
- Use parts of a circle (halves, fourths) and benchmark angle measurements (such as  $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ ,  $360^\circ$ ) to estimate the size of an angle, in degrees.

## Student Facing Learning Goals

Usemos grados para describir el tamaño de un ángulo.

## Lesson Purpose

The purpose of this lesson is for students to understand that angles can be measured in degrees and to use benchmark angle measurements to make sense of the new unit.

## Narrative

Previously, students used the hands of an analog clock to describe and compare the sizes of angles. In this lesson, students learn about degrees as a unit for measuring angles.

In the first activity, students are introduced to 360 degrees as the measurement of a full rotation of a ray about a fixed point. They use this to interpret and describe other benchmark angle measurements ( $90^\circ$ ,  $180^\circ$ ,  $270^\circ$ ). They then use these benchmarks to estimate and sketch new angles with given measurements in degrees.

Next, students use these reference angles to create an angle measurement tool from paper. They partition the straight angle of a semicircle into smaller angles by folding. In doing so, they draw from their experience with the clock, where each hour or each minute can be thought of as equal-size parts around the center point of the clock.

Throughout the lesson, listen for the way students make connections to their work with clocks, and to their understanding of fractions of a circle, as they reason about how to estimate and sketch angles in degrees, using an understanding that a full rotation is 360 degrees.

## Required Materials

### Materials to Gather

- Paper: Activity 2
- Rulers or straightedges: Activity 2

### Materials to Copy

- Making a Measuring Tool Template (1 copy for every 3 students): Activity 2



## Lesson Timeline

Warm-up	10 min
Activity 1	15 min
Activity 2	20 min
Synthesis Estimate	10 min
Actividad de cierre	5 min

## Teacher Reflection Questions

How did students connect the angles they created in the second activity to the fractions of a circle? How can you help students make connections between degrees and a fraction of a circle in upcoming lessons?

## Warm-up

 10 min

¿Qué sabes sobre el 360?

### Standards

Building On **4.NBT.A.1**  
Building Toward **4.MD.C.5, 4.MD.C.7**

### Instructional Routines

- What Do You Know about \_\_\_\_?

The purpose of this *Warm-up* is to invite students to think about 360 in terms of related numbers—as a result of addition, subtraction, multiplication, or division. The reasoning done here will be helpful when students compose angles into a sum of  $360^\circ$  or decompose a  $360^\circ$  angle into smaller angles, particularly benchmark angles such as  $30^\circ$ ,  $60^\circ$ ,  $90^\circ$ , and  $180^\circ$ .

### Student Task Statement

 ¿Qué sabes sobre el 360?

### Student Response

Sample responses:

- It is 40 less than 400, and 60 more than 300.
- It is 10 times 36, twice 180, and half of 720.
- It is a multiple of 10, 20, 30, 40, 60, 90, 120, 180.
- It is  $2 \times 180$ ,  $3 \times 120$ ,  $4 \times 90$ ,  $6 \times 60$ ,  $9 \times 40$ ,  $10 \times 36$ ,  $12 \times 30$ ,  $15 \times 24$ ,  $18 \times 20$ .

### Launch

- Display the number.
- “¿Qué saben sobre el 360?” // “What do you know about 360?”

### Activity

- 1 minute: quiet think time
- Record responses.
- If no students mentioned different ways to express 360, ask: “How could we express the number 360?” and “What do you know about the factors of 360?”

### Activity Synthesis

- Draw students’ attention to the factors of 360. “¿Cuáles son los factores de 360? ¿Cuántos hay?” // “What are the factors of 360? How many are there?”
- “El número 360 y sus factores son importantes cuando describimos ángulos. Descubramos por qué aparecen una y otra vez mientras estudiamos una



nueva manera de describir y medir el tamaño de un ángulo" // "The number 360 and its factors are important when describing angles. Let's find out why they show up again and again as we look at a new way to describe and measure the size of an angle."

## Activity 1

🕒 15 min

Un giro completo

### Standards

Addressing 4.MD.C.5.a, 4.MD.C.7

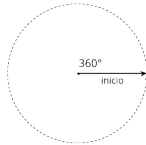
In previous activities, students used the features of a clock to describe and compare angles. This activity introduces students to the **degree** as a unit of measure.

Because one degree is much more abstract than one inch or one square-inch tile, students are first introduced to 360 degrees as a full rotation of a ray around its starting point. Students use this information to reason about other angle measurements. They may use their understanding of fractions of a circle to determine the sizes (MP7).

In the *Activity Synthesis*, students describe angle measurement as additive. Students will continue to build this understanding and reason about the size of 1 degree in subsequent lesson activities.

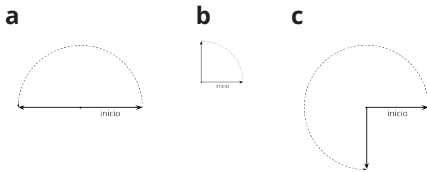
### Student Task Statement

Un rayo que da una vuelta entera alrededor de su punto de inicio ha dado un giro completo.



Decimos que el rayo giró 360 **grados**.

1. ¿Cuántos grados giró el rayo desde donde comenzó?



2. Dibuja 2 ángulos:
  - a. un ángulo en el que un rayo haya girado  $50^\circ$
  - b. un ángulo en el que un rayo haya girado  $130^\circ$

### Launch

- Groups of 2
- *"Así como la longitud y el área se pueden medir en unidades estándar, los ángulos también tienen unidades estándar" // "Just like length and area can be measured in standard units, angles have standard units."*
- *"Una unidad estándar para medir el tamaño de un ángulo es el **grado**" // "One standard unit for measuring the size of an angle is **degree**."*
- Display the image of the ray turning 360 degrees and read the opening *Task Statement*.

### Activity

- *"Usen esta información para encontrar cuántos grados giró cada rayo desde donde comenzó. Después, dibujen algunos ángulos que tengan aproximadamente el tamaño dado en grados" // "Use this information to determine how many degrees each ray has turned from where it started. Then sketch some angles that are about the size given in degrees."*



## Student Response

- 180 degrees
  - 90 degrees
  - 270 degrees
- Students' sketches need not be precise, but should show reasonable estimates of an angle that is between 40 degrees and 60 degrees for the 50-degree angle and between 120 degrees and 140 degrees for 130-degree angle.

- 3–4 minutes: independent work time
- *“Comparen con su compañero cómo pensaron. Expliquen cómo hicieron sus estimaciones”* // “Compare your thinking with your partner. Explain how you made your estimates.”
- 2–3 minutes: partner discussion
- Monitor for students who:
  - Describe the 180-degree turn as half a full turn or turning half a circle.
  - Describe the 90-degree angle as half of half a turn or as a turning a fourth of the way around the circle.
  - Use their estimate for the 90-degree angle to estimate the 270-degree angle, by adding ( $90 + 90 + 90$  or  $180 + 90$ ) or by subtracting ( $360 - 90$ ).

## Activity Synthesis

- Invite previously identified students to share how they estimated the turn of each ray in degrees.
- *“¿En qué se parecieron sus métodos? ¿En qué fueron diferentes?”* // “How were your methods the same? How were they different?” (We all used the first diagram to figure out how much of a turn. Some of us used one measure to find the next measure. Some of us thought about addition and some thought about division).
- Consider displaying the equation  $90 + 90 + 90 = 270$  and discussing:
  - *“¿Cómo corresponde esto a una manera de estimar el tercer ángulo?”* // “How does this match a way to estimate the third angle?”
  - *“¿Qué otra ecuación podemos usar para describir ese ángulo?”* // “What’s another equation we could use to describe that angle?”
- *“Un ángulo que mide 90 grados se llama un **ángulo recto**”* // “An angle that measures 90 degrees is called a **right angle**.”
- *“¿En dónde han visto ángulos rectos antes?”* // “Where have you seen right angles before?” (Corners of squares or rectangles. Corners of paper. The angle made by the hands of a clock when it is 3 o'clock or 9 o'clock.)



## Activity 2

🕒 20 min

Hagamos una herramienta para medir

### Standards

Addressing 4.MD.C.5.a, 4.MD.C.7

Building Toward 4.MD.C.6

In this activity, students construct a protractor-like tool that shows some benchmark angles. They do so by decomposing given angles— $120^\circ$  and  $180^\circ$ —and then looking for other ways to compose or decompose the subsequent angles identified along the way.

The activity serves several goals. The first is to familiarize students with the structure of a protractor, using tactile processes (folding paper and aligning lines or edges). The second goal is to develop students' intuition for thinking of a larger angle as composed of smaller angles, preparing them to see (in future lessons) that a  $1^\circ$  angle is  $\frac{1}{360}$  of a full turn. A final goal is to motivate the need for a tool that can measure angles more precisely.

Some students may use a square corner of a sheet of paper to find a  $90^\circ$  angle on their semicircle and others may choose to fold their semicircle in half. Expect most students to fold their paper to find all subsequent angles.

### Required Materials

#### Materials to Gather

- Paper: Activity 2
- Rulers or straightedges: Activity 2

#### Materials to Copy

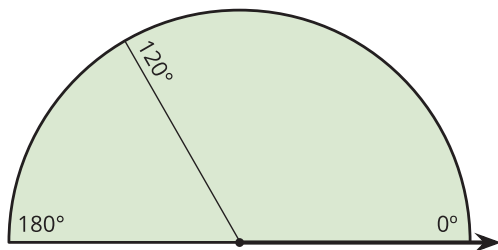
- Making a Measuring Tool Template (1 copy for every 3 students): Activity 2

### Required Preparation

- Create a paper half circle from the blackline master for each student.

### Student Task Statement

Tu profesor te va a dar una hoja de papel que tiene forma de medio círculo. En la hoja se muestra un ángulo de  $120^\circ$  y un ángulo de  $180^\circ$  medidos desde el rayo de la parte inferior derecha.



En la hoja de medio círculo:

1. Dibuja un segmento de recta que muestre un

### Launch

- Groups of 2–4
- Give each student one paper half-circle and access to rulers or straightedges.
- *“Su hoja de papel tiene la forma de medio círculo. La hoja muestra un rayo en la parte inferior derecha y dos ángulos ( $120^\circ$  y  $180^\circ$ ) medidos desde el rayo” // “Your sheet of paper is in the shape of half a circle. It shows a ray on the bottom right and two angles ( $120^\circ$  and  $180^\circ$ ) measured from the ray.”*
- *“Vemos la marca del  $120^\circ$ . ¿Dónde está el ángulo de  $120^\circ$ ? ¿Dónde están los dos rayos que forman este ángulo?” // “We see the  $120^\circ$  label. Where is the  $120^\circ$  angle? Where are the two rays that make this angle?”*
- 1 minute: quiet think time for the first problem
- 1 minute: group discussion



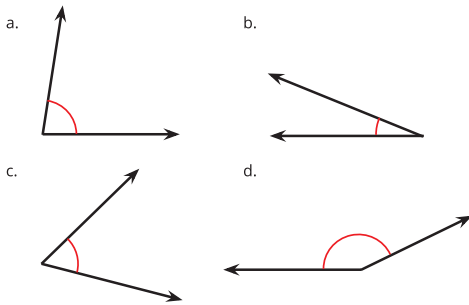
ángulo de  $90^\circ$  desde el mismo rayo. Márcalo con la medida. Intenta ser lo más preciso posible.

2. Dibuja segmentos de rectas que muestren los siguientes ángulos (medidos desde el mismo rayo). Marca cada segmento con la medida.

$60^\circ$     $45^\circ$     $30^\circ$     $150^\circ$     $135^\circ$

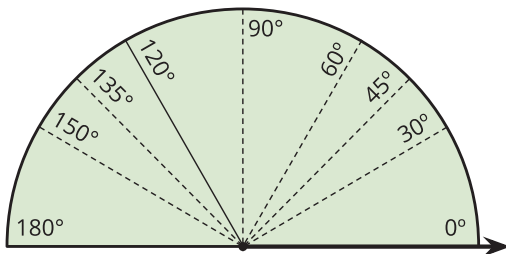
3. ¿Puedes encontrar un ángulo de  $1^\circ$  desde el mismo rayo? Explica o muestra cómo lo sabes.
4. ¡Hiciste una herramienta para medir!

¿Cómo se usa la herramienta para estimar el tamaño de un ángulo? Discute tus ideas con tu grupo. Después, usa la herramienta para estimar los tamaños de al menos 2 ángulos.



## Student Response

1. See image.
2. See image.



3. Sample responses:
  - a. Split  $30^\circ$  into three equal parts to get  $10^\circ$  each. Split the  $10^\circ$  into two parts of  $5^\circ$  each. Break each  $5^\circ$  angle into five parts of  $1^\circ$ .
  - b. It can't really be done—not precisely, anyway. The angle is too small to find by folding.
4. Sample responses:

- “¿Dónde piensan que estaría el segundo rayo de un ángulo de  $90^\circ$ ?” // “Where do you think the second ray of a  $90^\circ$  angle would be?” (Between  $0^\circ$  and  $120^\circ$ , but closer to  $120^\circ$ .)

## Activity

- 5–7 minutes: independent work time
- As students work on the last problem, monitor their ideas for using their tool to estimate angle measurements.

## Activity Synthesis

- “¿Cómo encontraron un ángulo de  $90^\circ$ ?” // “How did you find a  $90^\circ$  angle?” (I folded the semicircle into halves.)
- “¿Cómo encontraron todos los demás ángulos?” // “How did you find all the other angles?” (For  $60^\circ$ , fold to line up the thick ray with the  $120^\circ$  segment, splitting 120 into two. For  $45^\circ$ , fold to line up the ray with the  $90^\circ$  segment, splitting 90 into two. Repeat in a similar fashion to find the others.)
- Invite students to share how they might find a  $1^\circ$  angle in their half circle. Highlight explanations that involve finding some fraction of smaller and smaller angles.
- Solicit some estimates of the angle measurements in the last problem. Record and display them for comparison later, when the same four angles are measured, using a protractor.
- Students likely will notice that their tool is imprecise and not reliable or practical for measuring angles beyond estimations. Explain that we will look at another tool in the next activity.

- a.  $80^\circ$
- b.  $25^\circ$
- c.  $60^\circ$
- d.  $155^\circ$

## Advancing Student Thinking

If students create angles by estimating the locations of the lines, consider asking:

- “¿Cómo decidiste en dónde crear los ángulos?” // “How did you decide where to create the angles?”
- “¿Crees que el ángulo que necesitas crear es más pequeño o más grande que los que ya dibujaste? Si hicieras dobles, ¿cómo te ayudaría eso a crear ángulos más precisos en esta actividad?” // “Do you think the angle you need to create is smaller or larger than those that you have drawn already? How might folding help you create more precise angles during this task?”

## Lesson Synthesis

“En lecciones anteriores, usamos relojes para ayudarnos a comparar ángulos y para hablar sobre su tamaño. Hoy aprendimos que los ángulos se pueden medir en grados y que algunos ángulos son útiles para estimar el tamaño de otros ángulos” // “In earlier lessons, we used clocks to help us compare angles and talk about their sizes. Today we learned that angles can be measured in degrees and that some angles are helpful for estimating the sizes of other angles.”

“¿Qué aprendieron sobre  $360^\circ$ ?” // “What did you learn about  $360^\circ$ ?” (It’s the measurement of the angle made by a ray that makes a full turn around a point.)

“¿Cómo describirían un ángulo de  $180^\circ$ ?” // “How would you describe a  $180^\circ$  angle?” (It’s half of a full turn. The two rays make a straight line or point in opposite directions.)

“¿Cómo describirían un ángulo de  $90^\circ$ ?” // “How would you describe a  $90^\circ$  angle?” (It’s a half of a half turn. It’s the size of an angle made when a ray makes a fourth of a full turn around a point. It is called a **right angle**. It’s the size of the angle at the corner of a piece of paper.)

## Suggested Centers

- Compare (1–5), Stage 6: Fractions (Supporting)
- Target Measurements (2–5), Stage 4: Degrees (Addressing)

## Cool-down

 5 min

Estima el tamaño del ángulo en grados

### Standards

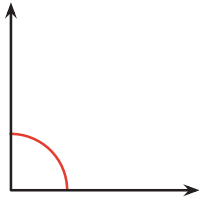
Addressing 4.MD.C.5.a



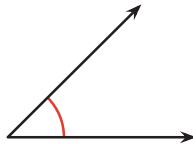
## Student Task Statement

Usa la herramienta que hiciste para estimar el tamaño de cada ángulo, en grados.

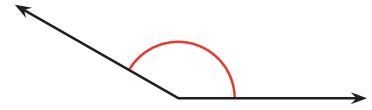
a



b



c



## Student Response

1. 90 degrees
2. 45 degrees
3. 150 degrees

## Responding to Student Thinking

Students find angle measurements that are more than 5 degrees greater than or less than  $90^\circ$ ,  $45^\circ$ , or  $150^\circ$ .

Next Day Supports

Launch Activity 1, with a discussion about this *Cool-down*.