Graphs of Rational Functions (Part 2)

Let's learn about horizontal asymptotes.

3.1 Rewritten Equations

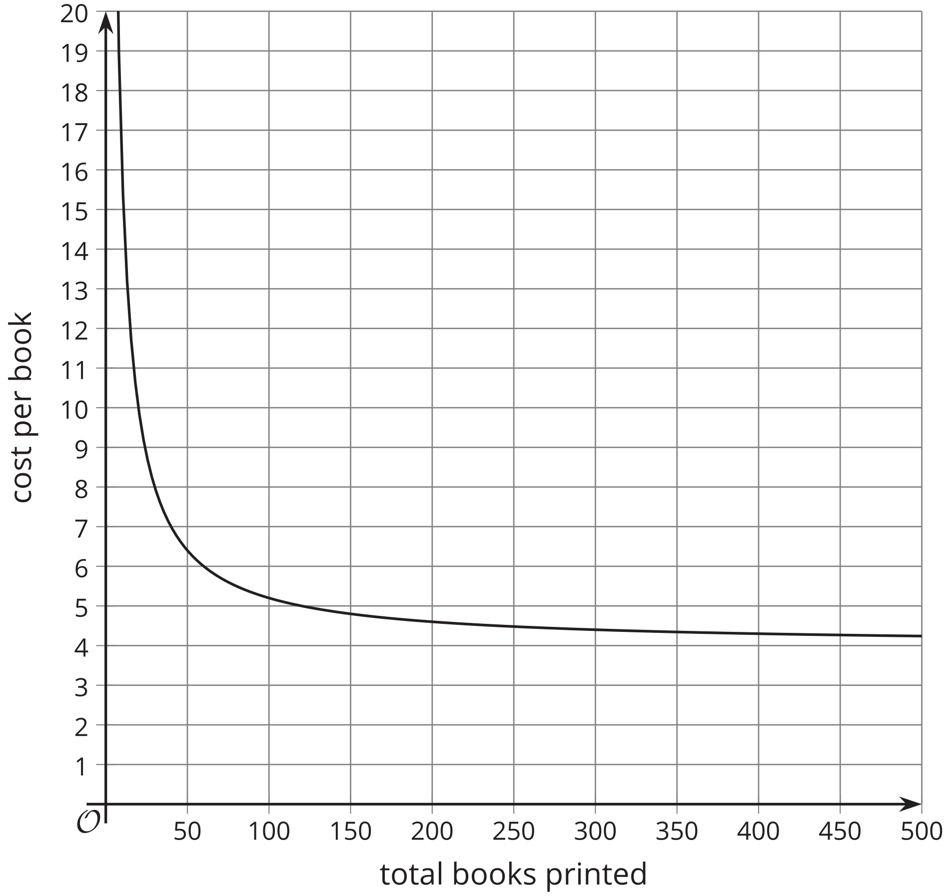
Decide if each of these equations is true or false for x-values that do not result in a denominator of 0. Be prepared to explain your reasoning.

1. $\frac{x+7}{x} = 1 + \frac{7}{x}$

2. $\frac{x}{x+7} = 1 + \frac{x}{7}$

3.2 Publishing a Paperback

Let c be the function that gives the average cost per book $c(x)$, in dollars, when using an online store to print x copies of a self-published paperback book. Here is a graph of $c(x) = \frac{120+4x}{x}$.



1. What is the approximate cost per book when 50 books are printed? 100 books?

2. The author plans to charge \$8 per book. About how many should be printed to make a profit?

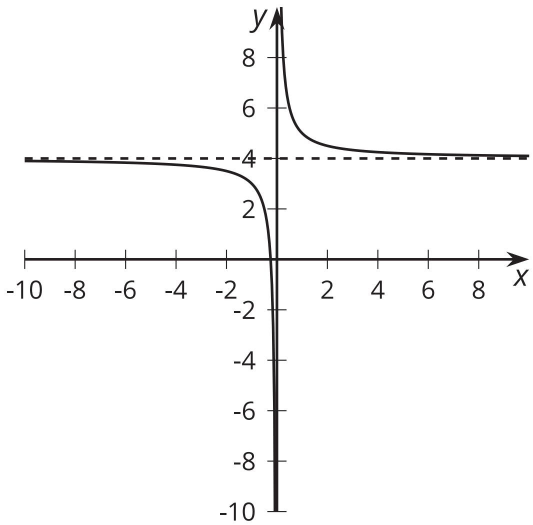
3. What is the value of $c(x)$ when $x = \frac{1}{2}$? How does this relate to the context?

4. What does the end behavior of the function say about the context?

3.3 Horizontal Asymptotes

Here are four graphs of rational functions.

A



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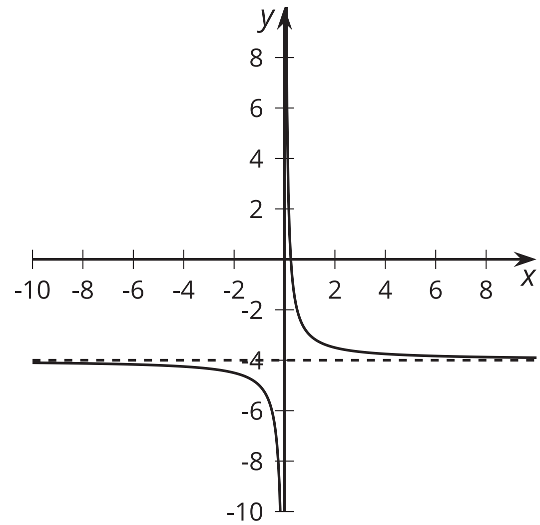
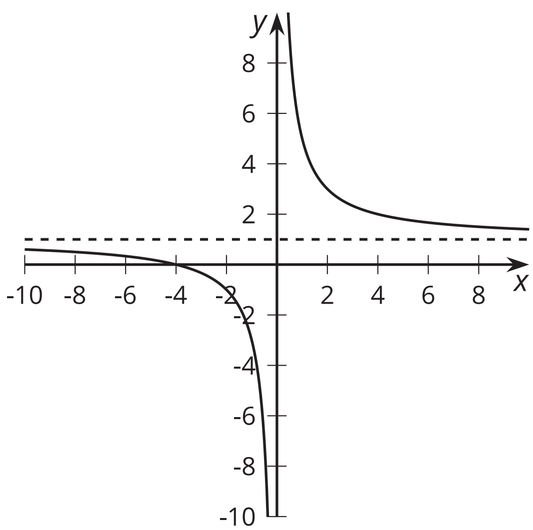


C



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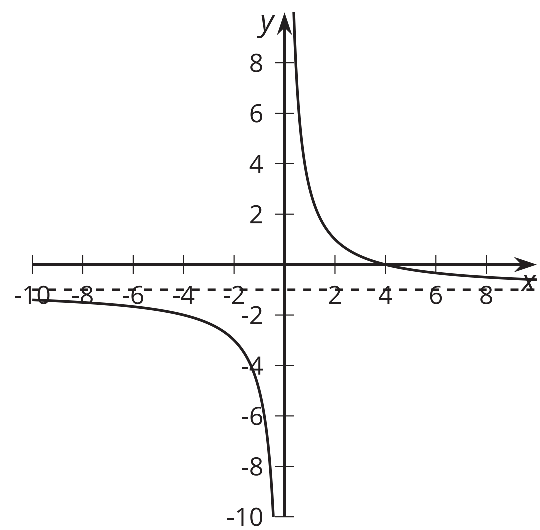


1. Match each function with its graphical representation.
   - $a(x) = \frac{4}{x} - 1$
   - $b(x) = \frac{1}{x} - 4$
   - $c(x) = \frac{1+4x}{x}$
   - $d(x) = \frac{x+4}{x}$
   - $e(x) = \frac{1-4x}{x}$
   - $f(x) = \frac{4-x}{x}$
   - $g(x) = 1 + \frac{4}{x}$
   - $h(x) = \frac{1}{x} + 4$

2. Where do you see the **horizontal asymptote** of the graph in the expressions for the functions?

Are you ready for more?

Consider the function $a(x) = \frac{\frac{1}{2}x+1}{x-1}$.

1. Predict where you think the vertical and horizontal asymptotes of $a(x)$ will be. Explain your reasoning.

2. Use graphing technology to check your prediction.

Lesson 3 Summary

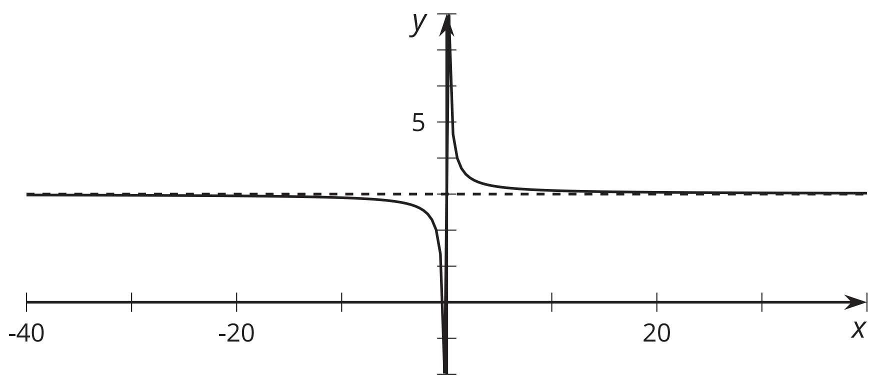
Consider the rational function $f(x) = \frac{3x+1}{x}$. Written this way, we can tell that the graph of the function has a vertical asymptote at $x = 0$ by reading the denominator and identifying the value that would cause division by 0. But what can we tell about the value of $f(x)$ for values of x far away from the vertical asymptote?

One way we can think about these values is to rewrite the expression for $f(x)$ by breaking up the fraction:

$f(x) = \frac{3x}{x} + \frac{1}{x}$

$f(x) = 3 + \frac{1}{x}$

Written this way, it's easier to see that as x gets larger and larger in either the positive or negative direction, the $\frac{1}{x}$ term will get closer and closer to 0. Because of this, we can say that the value of the function will get closer and closer to 3. Here is a graph of $y = f(x)$ showing values from -40 to 40.



A dashed line at $y = 3$ is included to show how the function approaches this value as inputs are farther and farther from $x = 0$. This is an example of a feature of rational functions: a horizontal asymptote.

The line $y = c$ is a **horizontal asymptote** for a function if the value of the function gets closer and closer to c as the magnitude of x increases.

More generally, if a rational function $g(x) = \frac{a(x)}{b(x)}$ can be rewritten as $g(x) = c + \frac{r(x)}{b(x)}$, where c is a constant and $r(x)$ and $b(x)$ are polynomial expressions in which $\frac{r(x)}{b(x)}$ gets closer and closer to 0 as x gets larger and larger in both the positive and negative directions, then $g(x)$ will get closer and closer to c.

