Unit 1, Lesson 5

Bases and Heights of Parallelograms

Goals

- Comprehend that the terms “base” and “height” refer to one side of a parallelogram and the perpendicular distance between that side and the opposite side.
- Generalize (orally) a process for finding the area of a parallelogram, using the length of a base and the corresponding height.
- Identify a base and the corresponding height for a parallelogram, and understand that there are two different base-height pairs for any parallelogram.

Learning Targets

- I can identify pairs of base and height of a parallelogram.
- I can write and explain the formula for the area of a parallelogram.
- I know what the terms "base" and "height" refer to in a parallelogram.

Lesson Narrative

In this lesson, students learn about **bases** and **heights** of a parallelogram and generalize the process for finding the area of a parallelogram.

Students begin by comparing two strategies for finding the area of a parallelogram. This comparison sets the stage for seeing a rectangle that is associated with a parallelogram and for understanding bases and heights.

Next, students examine examples and non-examples of base-height pairs to make sense of the terms “base” and “height” in a parallelogram. They also practice identifying the corresponding height for a given base.

Then students identify both a base and its corresponding height for given parallelograms and find their areas. Through repeated reasoning, students notice regularity in the process of finding the area of a parallelogram and express it as a formula in terms of base and height (MP8).

A note about terminology:

The terms “base” and “height” are potentially confusing because they are sometimes used to refer to particular line segments, and sometimes to the length of a line segment or the distance between parallel lines. Furthermore, there are always two base-height pairs for any parallelogram, so asking for the base and the height is not, technically, a well-posed question. Instead, asking for a base and its corresponding height is more appropriate. In these materials, the words “base” and “height” mean the numbers unless it is clear from the context that it means a segment and that there is no potential confusion.

A note about notation:

In this lesson, students see the “dot” notation for multiplication in their materials for the first time. If needed, reiterate that both the $\cdot$ symbol and the $\times$ symbol represent multiplication.

Math Community
Today’s community building centers on the teacher sharing their draft commitments as part of the mathematical community. At the end of the lesson, students are invited to suggest additions to the teacher sections of the chart.

Standards

Addressing **6.EE.A.2.a, 6.EE.A.2.c, 6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Materials to Gather

- Geometry toolkits: Lesson, Activity 1, Activity 3
- Math Community Chart: Activity 1, Cool-down

Required Preparation

Activity 1:

For the digital version of the activity, acquire devices that can run the applet.

In the "Doing Math" teacher section of the Math Community Chart, add 2–5 commitments you have for what your teaching practice "looks like" and "sounds like" this year.

Activity 2:

For the digital version of the activity, acquire devices that can run the applet.

Student Facing Learning Goals

Let's investigate the area of parallelograms some more.

5.1 A Parallelogram and Its Rectangles

Warm-up

10 min

Activity Narrative

There is a digital version of this activity.

In this *Warm-up*, students compare and contrast two ways of decomposing and rearranging a parallelogram on a grid such that its area can be found. This work allows students to practice communicating their observations and prompts them to notice features of a parallelogram that are useful for finding area—a base and its corresponding height.

The flow of key ideas—to be uncovered during discussion and gradually throughout the lesson—is as follows:

- There are multiple ways to decompose a parallelogram (with one cut) and rearrange it into a rectangle whose area we can determine.
- The cut can be made in different places, but to compose a rectangle, the cut has to be at a *right angle* to two opposite sides of the parallelogram.
- The length of one side of this newly composed rectangle is the same as the length of one side of the parallelogram. We use the term **base** to refer to this side.
- The length of the other side of the rectangle is the length of the cut we made to the parallelogram. We call this segment a **height** that corresponds to the chosen base.
- We use these two lengths to determine the area of the rectangle, and thus also the area of the parallelogram.

As students work and discuss, identify those who recognize that both Elena and Tyler decomposed the parallelogram by making a cut that is perpendicular to one side and then rearranged the pieces into a rectangle. Ask them to share their observations later. Be sure to leave enough time to discuss the first four key ideas as a class.

In the digital version of the warm-up, students use applets to animate the moves that Elena and Tyler made (decomposing and rearranging) to find the area of the parallelogram.

Standards

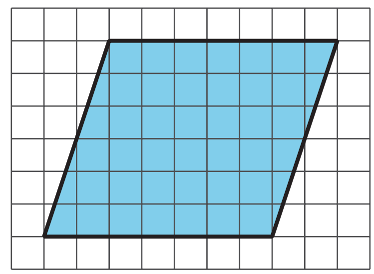
Addressing **6.G.A.1**

Launch

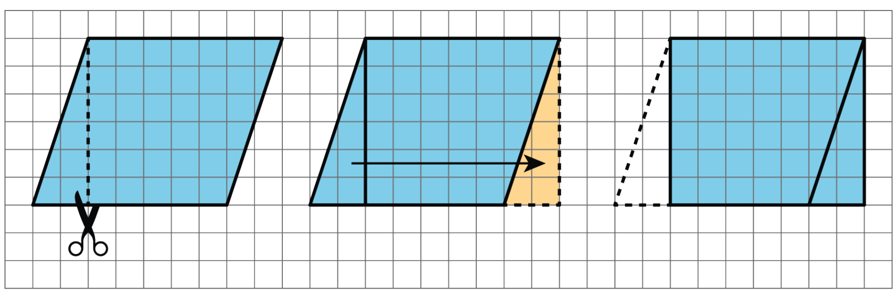
Arrange students in groups of 2. Give students 2 minutes of quiet think time and access to geometry toolkits. Ask them to share their responses with a partner afterward.

Student Task Statement

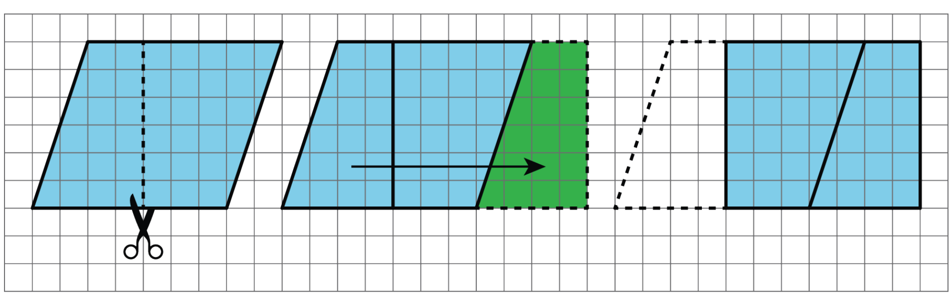
Elena and Tyler were finding the area of this parallelogram:



Here is how Elena did it:



Here is how Tyler did it:



How are the two strategies for finding the area of a parallelogram the same? How they are different?

Student Response

Sample responses:

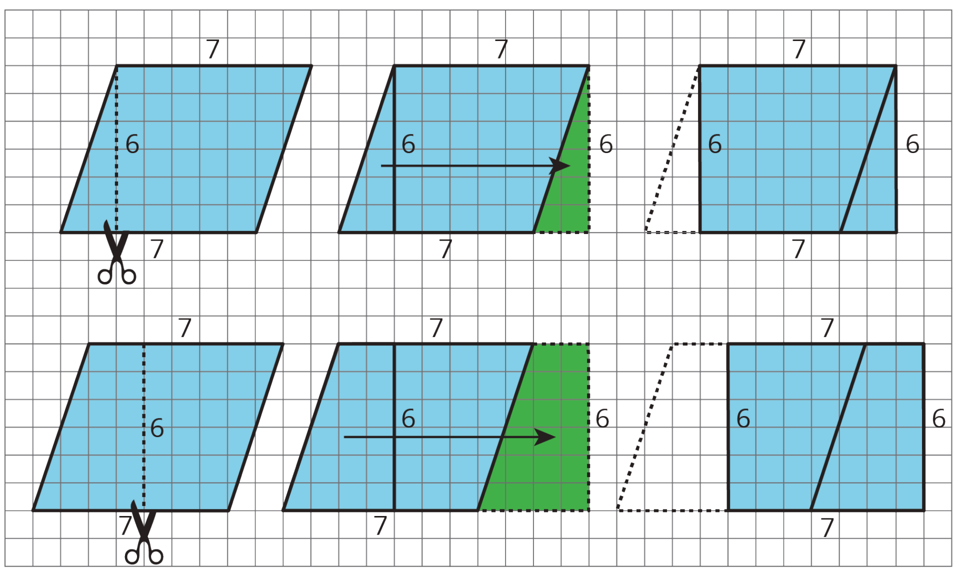
- Similar: They both cut off a piece from the left of the parallelogram and moved it over to the right to make a rectangle. The rectangles they made are identical.
- Different: They cut the parallelogram at different places. Elena cut a right triangle from the left side and Tyler cut off a trapezoid. The rectangles they made are not in the same place.

Activity Synthesis

Select previously identified students to share what was the same and what was different about Elena's and Tyler's methods.

If not already mentioned by students, highlight the following points on how Elena's and Tyler's approaches are the same, though do not expect students to use the language as written here. Clarify each point by gesturing, pointing, and annotating the images.

- The rectangles are identical. They have the same side lengths. (Label the side lengths of the rectangles.)
- The cuts were made in different places, but the length of the cuts was the same. (Label the lengths along the vertical cuts.)
- The horizontal sides of the parallelogram have the same length as the horizontal sides of the rectangle. (Point out how both segments have the same length.)
- The length of each cut is the distance between the two horizontal sides of the parallelogram. It is also the vertical side length of the rectangle. (Point out how that distance stays the same across the horizontal length of the parallelogram.)



Begin to connect the observations to the terms "**base**" and "**height**." For example, explain:

- "The two measurements that we see here have special names. The length of one side of the parallelogram—which is also the length of one side of the rectangle—is called a *base*. The length of the vertical cut segment—which is also the length of the vertical side of the rectangle—is called a *height* that corresponds to that base."
- "Here, the side of the parallelogram that is 7 units long is also called a base. In other words, the word 'base' is used for both the segment and the measurement."

Tell students that we will explore bases and heights of a parallelogram in this lesson.

Math Community

After the *Warm-up*, display the Math Community Chart with the "doing math" actions added to the teacher section for all to see. Give students 1 minute to review. Then share 2–3 key points from the teacher section and your reasoning for adding them. For example,

- If "questioning vs. telling," a shared reason could focus on your belief that students are capable mathematical thinkers and your desire to understand how students are making meaning of the mathematics.
- If "listening," a shared reason could be that sometimes you want to sit quietly with a group just to listen and hear student thinking and not because you think the group needs help or is off-track.

After sharing, tell students that they will have the opportunity to suggest additions to the teacher section during the *Cool-down*.

5.2 The Right Height?

15 min

Activity Narrative

There is a digital version of this activity.

In this activity, students further develop their understanding of bases and heights of parallelograms by studying examples and non-examples and by analyzing statements. The goal is for students to see that in a parallelogram:

- The term "base" refers to the length of one side and "height" to the length of a perpendicular segment between that side and the opposite side.
- Any side of a parallelogram can be a base.
- There are always two base-height pairs for a given parallelogram.

In the digital version of *Are You Ready for More?*, students use an applet to create a dynamic parallelogram with a height displayed. The applet allows students to see placements of height segments in a variety of parallelograms and when any side is chosen as a base. Consider allowing students to use the applet to check their responses to the last question (about whether the bases and heights in parallelograms A–E are correctly labeled) and to gain additional insights about base-height pairs.

Standards

Addressing **6.G.A.1**

Instructional Routines

- MLR8: Discussion Supports

Launch

Display the examples and non-examples of bases and heights for all to see. Read aloud the first paragraph of the activity and the description of each set of images. Give students a minute to observe the images. Then tell students to use the examples and non-examples to determine what is true about bases and heights in a parallelogram.

Arrange students in groups of 2. Give students 4–5 minutes to complete the first question with their partner. Ask them to pause for a class discussion after the first question. Select a student or a group to make a case for whether each statement is true or false. If one or more students disagree, ask them to explain their reasoning and discuss to reach a consensus. Before moving on to the next question, be sure students record the verified true statements so that they can be used as a reference later.

Give students 3 minutes of quiet time to answer the second question and another 2–3 minutes to share their responses with a partner. Ask them to focus partner conversations on the following questions, displayed for all to see:

- How do you know the parallelogram is labeled correctly or incorrectly?
- Is there another way a base and height could be labeled on this parallelogram?

After answering the questions, students with digital access can explore the applet ggbm.at/UnfbrN96 and use it to verify their responses and further their understanding of bases and heights.

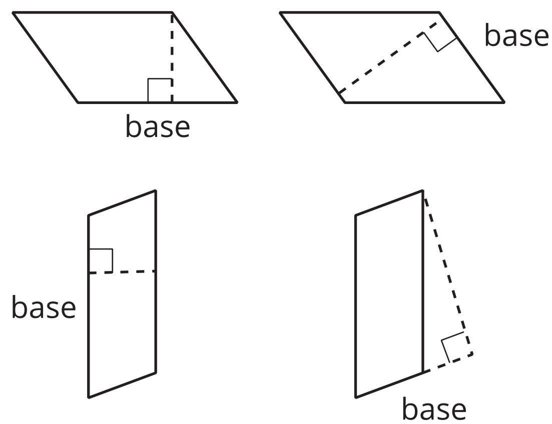
Access for English Language Learners

MLR8 Discussion Supports. Pair gestures with verbal directions to clarify the meaning of any unfamiliar terms such as "dashed," "horizontal," "opposite," or "perpendicular." *Advances: Listening, Representing*

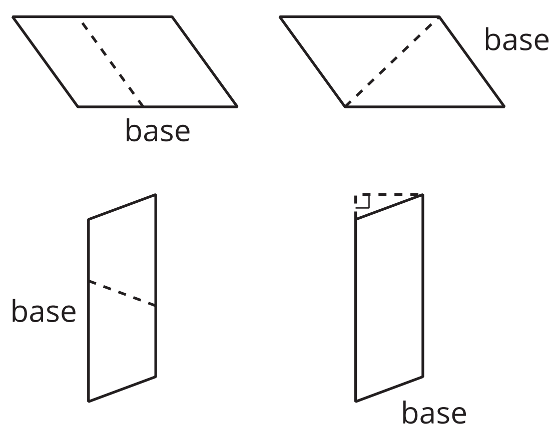
Student Task Statement

Here are some drawings of parallelograms. In each drawing, one side is labeled "**base**."

In the first four drawings, each dashed segment represents a **height** that corresponds to the given base.



In the next four drawings, each dashed segment does not represent a height that corresponds to the given base.



1. Select **all** the statements that are true about bases and heights in a parallelogram.

   A. Only a horizontal side of a parallelogram can be a base.

   B. Any side of a parallelogram can be a base.

   C. A height can be drawn at any angle to the side chosen as the base.

   D. A base and its corresponding height must be perpendicular to each other.

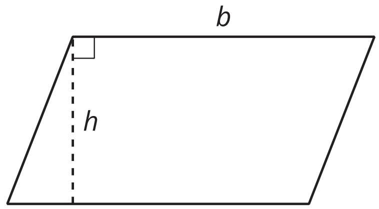
E. A height can only be drawn inside a parallelogram.

F. A height can be drawn outside of the parallelogram, as long as it is drawn at a 90-degree angle to the base.

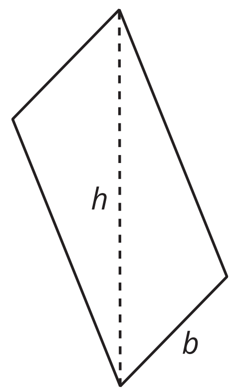
G. A base cannot be extended to meet a height.

2. Five students labeled a base b and a corresponding height h for each of these parallelograms. Are all drawings correctly labeled? Explain how you know.

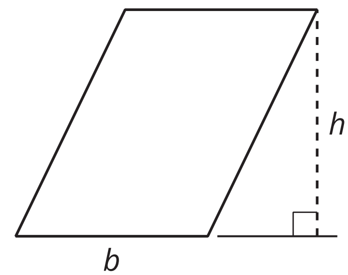
A



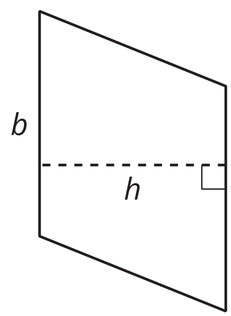
B



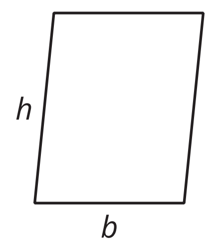
C



D



E



Student Response

1. Statements B, D, and F are true.
2. A, C, and D are correct. Sample reasoning: B and E are not correct because in each, the segment labeled with an h is not perpendicular to the side labeled with a b.

Building on Student Thinking

Students might say that Parallelogram E is correctly labeled because the labeled sides remind them of the labeled length and width of a rectangle. Ask students to revisit the true statements about base-height pairs and see if those conditions are met in Parallelogram E.

Activity Synthesis

Ask the class to give a quick agree-or-disagree signal on whether each figure in the last question is labeled correctly. After getting the responses for each figure, ask a student to explain how they know it is correct or incorrect.

If a parallelogram is incorrectly labeled, ask students where a correct height could be. If it is correctly labeled, ask students if there is another base and height that could be labeled on this parallelogram.

Before moving forward in this lesson, be sure that students understand which parallelograms are labeled correctly and emphasize the following points:

- We can choose any side of a parallelogram as a base.
- To find the height that corresponds to that base, draw a segment that joins the base and its opposite side. That segment has to be perpendicular to both the base and the opposite side.

Consider using the applet ggbm.at/UnfbrN96 to further illustrate possible base-height pairs and reinforce students' understanding of them.

Access for Students with Disabilities

Representation: Internalize Comprehension. Invite students to identify which details were most important in finding base-height pairs to solve the problem. Display the sentence frame, "The next time I need to find the base and height of a parallelogram, I will look for . . ."
Supports accessibility for: Language

5.3 Finding the Formula for Area of Parallelograms

10 min

Activity Narrative

In this activity, students find the area of more parallelograms, generalize the process, and write an expression for finding the area of any parallelogram. To do so, they apply what they learned in previous lessons about base-height pairs in parallelograms and about strategies for reasoning about area.

As students discuss their work, monitor conversations for any disagreements between partners. Support them by asking clarifying questions:

- "How did you choose a base? How can you be sure that is the height?"
- "How did you find the area? Why did you choose that strategy for this parallelogram?"
- "Is there another way to find the area and to check your answer?"

Standards

Addressing **6.EE.A.2.a, 6.G.A.1**

Launch

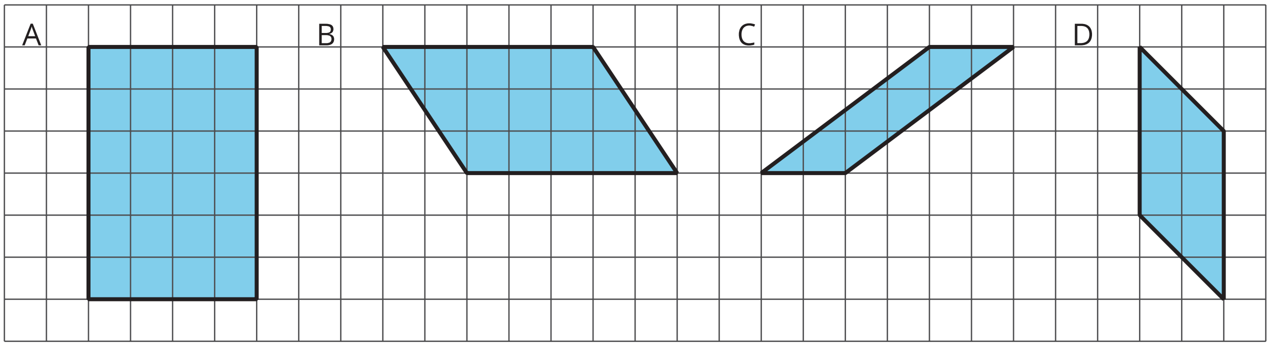
Keep students in groups of 2. Give students access to their geometry toolkits and 4–5 minutes of partner work time to complete the table. Ask them to be prepared to share their reasoning. Encourage students to use their work from earlier activities (on bases and heights) as a reference.

Student Task Statement

For each parallelogram:

- Identify a base and a corresponding height, and record their lengths in the table.

- Find the area of the parallelogram and record it in the last column of the table.



parallelogram	base (units)	height (units)	area (sq units)
A			
B			
C			
D			
any parallelogram	b	h	

In the last row of the table, write an expression for the area of any parallelogram, using b and h .

Student Response

While there are two possible base-height pairs, these are the easiest ones for students to use given the orientation of each parallelogram on the grid.

parallelogram	base (units)	height (units)	area (square units)
A	6 (or 4)	4 (or 6)	24
B	5	3	15
C	2	3	6
D	4	2	8
any parallelogram	b	h	$b \cdot h$

Building on Student Thinking

Finding a height segment outside of the parallelogram may still be unfamiliar to students. Have examples from the “The Right Height?” activity visible so they can serve as a reference in finding heights.

Students may say that the base of Parallelogram D cannot be determined because, as displayed, it does not have a horizontal side. Remind students that in an earlier activity we learned that any side of a parallelogram could be a base and that rotating our paper can help us see this. Ask students to see if there is a side whose length can be determined.

Are You Ready for More?

1. What happens to the area of a parallelogram if the height doubles but the base is unchanged? If the height triples? If the height is 100 times the original?
2. What happens to the area if both the base and the height double? Both triple? Both are 100 times their original lengths?

Extension Student Response

1. The area doubles, triples, is multiplied by 100.
2. The area quadruples, is 9 times the original area, is 10,000 times the original area.

Activity Synthesis

Display the parallelograms and the table for all to see. Select a few students to share the correct answers for each parallelogram. As students share, highlight the base-height pairs on each parallelogram and record the responses in the table. Although only one base-height pair is named for each parallelogram, reiterate that there is another pair. Show the second pair on the diagram or ask students to point it out.

After the first four rows of the table are completed, discuss the expression in the last row. Ask students:

- "How did you figure out the expression for the area for any parallelogram?" (The areas of Parallelograms A–D are each the product of base and height.)
- "Suppose you decompose a parallelogram with a cut and rearrange it into a rectangle. Does this expression for finding area still work? Why or why not?" (Yes. One side of the rectangle will have the same length as the base of the parallelogram. The height of the parallelogram is also the height of the rectangle—both are perpendicular to the base.)

Be sure everyone has the correct expression for finding the area of a parallelogram by the end of the discussion.

Lesson Synthesis

In this lesson, students identified a *base* and a corresponding *height* in a parallelogram, and then wrote an algebraic expression for finding the area of any parallelogram. Consider asking students:

- "How do you identify the base of a parallelogram?"(Any side can be a base. Sometimes one side is preferable over another because its length is known or easy to know.)
- "Once we have chosen a base, how can we identify a height that corresponds to it?" (Identify a perpendicular segment that connects that base and the opposite side; find the length of that segment.)
- "In how many ways can we identify a base and a height for a given parallelogram?" (There are two possible bases. For each base, many possible segments can represent the corresponding height.)
- "What is the relationship between the base and height of a parallelogram and its area?" (The area is the product of base and height.)

If time permits, ask students: "Do you think this expression will always work?" Students are not expected to prove their answer here. Speculation is expected at this point. The question is intended to prompt students to think of other differently-shaped parallelograms beyond the four shown here.

5.4 Parallelograms S and T

Cool-down

5 min

Standards

Addressing **6.EE.A.2.c, 6.G.A.1**

Launch

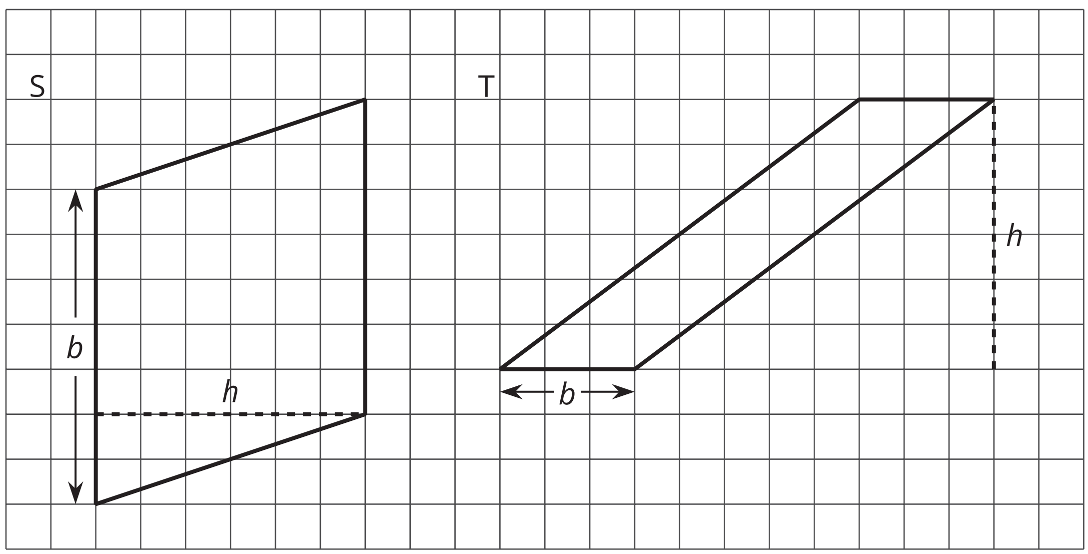
Math Community

Before distributing the *Cool-downs*, display the Math Community Chart and the community building question "What additions would you make to the teacher 'Doing Math' section of the Math Community Chart?" Ask students to respond to the question after completing the *Cool-down* on the same sheet.

After collecting the *Cool-downs*, identify themes from the community building question. Use them to add to or revise the teacher "Doing Math" section of the Math Community Chart before Exercise 4.

Student Task Statement

Parallelograms S and T are each labeled with a base and a corresponding height.



1. What are the values of b and h for each parallelogram?
   - Parallelogram S: b = ________, h = ________
   - Parallelogram T: b = ________, h = ________
2. Use the values of b and h to find the area of each parallelogram.
   - Area of Parallelogram S:
   - Area of Parallelogram T:

Student Response

1. - Parallelogram S: $b = 7$, $h = 6$

- Parallelogram T: $b = 3$, $h = 6$

2.
   - Area of Parallelogram S: 42 square units. $7 \cdot 6 = 42$
   - Area of Parallelogram T: 18 square units. $3 \cdot 6 = 18$

Responding to Student Thinking

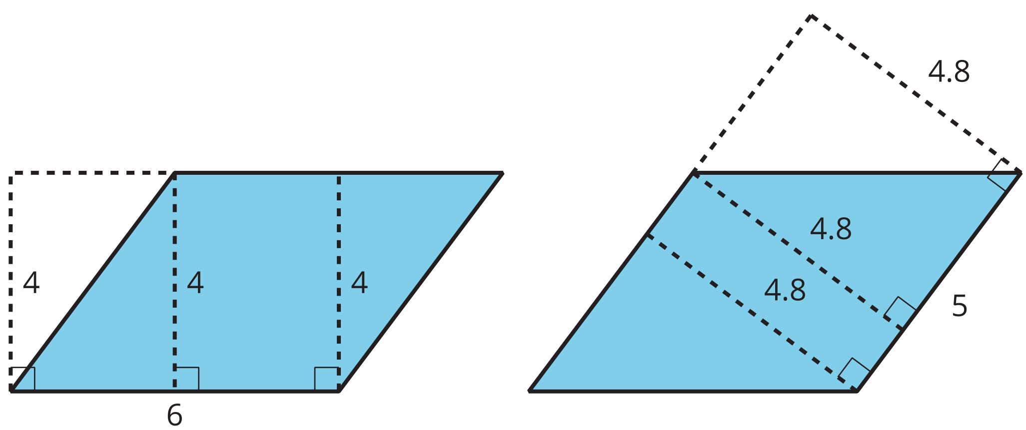
Points to Emphasize
If students struggle with finding the areas of the parallelograms, highlight ways to decompose and rearrange a parallelogram into a rectangle with a known base and height. For example, in this activity, emphasize that each parallelogram can be decomposed and rearranged into a rectangle with the same pair of base and height measurements:

Grade 6, Unit 1, Lesson 6, Activity 2 More Areas of Parallelograms

Lesson 5 Summary

- We can choose any side of a parallelogram as the **base**. Both the side selected (the segment) and its length (the measurement) are called the base.
- If we draw any perpendicular segment from a point on the base to the opposite side of the parallelogram, that segment will always have the same length. We call that value the **height**. There are infinitely many segments that can represent the height!

Here are two copies of the same parallelogram.



On the left, the side that is the base is 6 units long. Its corresponding height is 4 units.

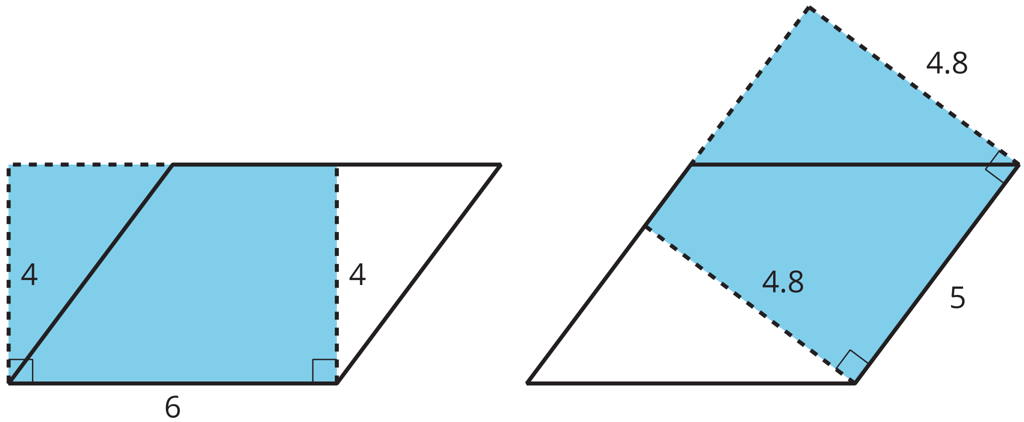
On the right, the side that is the base is 5 units long. Its corresponding height is 4.8 units.

For both, three different segments are shown to represent the height. We could draw in many more!

No matter which side is chosen as the base, the area of the parallelogram is the product of that base and its corresponding height. We can check this:

$4 \times 6 = 24$ and $4.8 \times 5 = 24$

We can see why this is true by decomposing and rearranging the parallelograms into rectangles.



Notice that the side lengths of each rectangle are the base and height of the parallelogram. Even though the two rectangles have different side lengths, the products of the side lengths are equal, so they have the same area! And both rectangles have the same area as does the parallelogram.

We often use letters to stand for numbers. If b is a base of a parallelogram (in units), and h is the corresponding height (in units), then the area of the parallelogram (in square units) is the product of these two numbers:

$$b \cdot h$$

Notice that we write the multiplication symbol with a small dot instead of a $\times$ symbol. This is so that we don't get confused about whether $\times$ means multiply, or whether the letter x is standing in for a number.

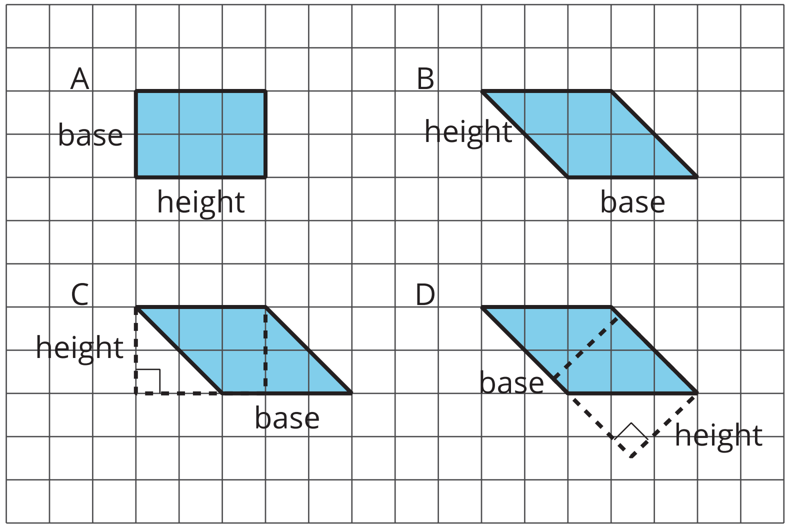
Glossary

- base (of a parallelogram or triangle)
- height (of a parallelogram or triangle)

Lesson 5 Practice Problems

1 Student Task Statement

Select **all** parallelograms that have a correct height labeled for the given base.



A. A

B. B

C. C

D. D

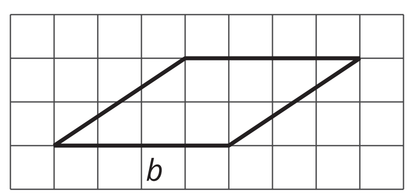
Solution

A, C, D

2 Student Task Statement

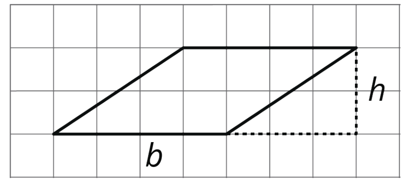
The side labeled b has been chosen as the base for this parallelogram.

Draw a segment showing the height corresponding to that base.



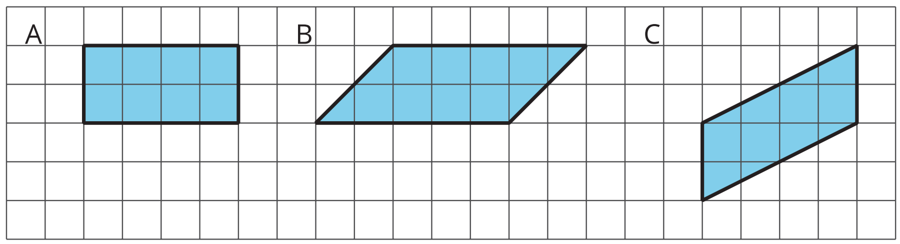
Solution

The height can be any segment perpendicular to the base that joins the line containing the base to the line containing the side opposite the base. Sample response:



3 Student Task Statement

Find the area of each parallelogram.



Solution

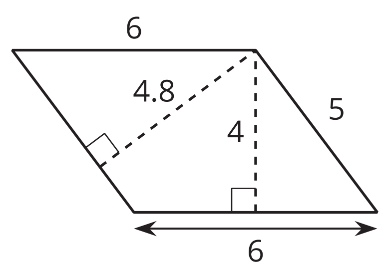
A: 8 square units. This is a 2-by-4 rectangle.

B: 10 square units. The horizontal side is 5 units long and can be the base. The height for this base is 2 units.

C: 8 square units. The vertical side can be used as the base. The base is 2 units, and the height for this base is 4 units.

4 Student Task Statement

If the side that is 6 units long is the base of this parallelogram, what is its corresponding height?



A. 6 units

B. 4.8 units

C. 4 units

D. 5 units

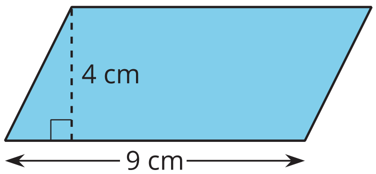
Solution

C

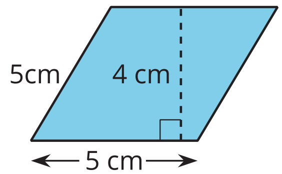
5 Student Task Statement

Find the area of each parallelogram.

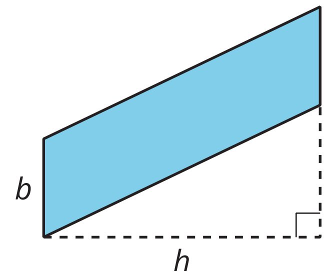
A



B



C



Solution

A: 36 sq cm. The base is 9 cm, and the height for that base is 4 cm.

B: 20 sq cm. The base is 5 cm, and the height for this base is 4 cm.

C: bh. The base is b, and the corresponding height is h.

6

from Unit 1, Lesson 4

Student Task Statement

Do you agree with each of these statements? Explain your reasoning.

a. A parallelogram has six sides.
b. Opposite sides of a parallelogram are parallel.
c. A parallelogram can have one pair or two pairs of parallel sides.
d. All sides of a parallelogram have the same length.
e. All angles of a parallelogram have the same measure.

Solution

a. Disagree. Sample reasoning: A parallelogram is a quadrilateral.
b. Agree. Sample reasoning: By definition, opposite sides of a parallelogram are parallel.
c. Disagree. Sample reasoning: By definition, a parallelogram has two pairs of parallel sides.
d. Disagree. Sample reasoning: Sometimes all sides of a parallelogram have the same length, but not always. Opposite sides of a parallelogram always have the same length.

e. Disagree. Sample reasoning: Sometimes all angles of a parallelogram have the same measure (when the parallelogram is a rectangle), but not always. Opposite angles of a parallelogram have the same measure.

7 from Unit 1, Lesson 2

Student Task Statement

A square with an area of 1 square meter is decomposed into 9 identical smaller squares. Each smaller square is decomposed into two identical triangles.

a. What is the area, in square meters, of 6 triangles? If you get stuck, consider drawing a diagram.

b. How many triangles are needed to compose a region that is $1\frac{1}{2}$ square meters?

Solution

a. $\frac{6}{18}$ or $\frac{1}{3}$ square meter.

b. 27 triangles. It takes 18 triangles to make an area of 1 square meter and 9 triangles to make an area of $\frac{1}{2}$ square meter. $18 + 9 = 27$.

8 from an earlier course

Student Task Statement

Which shape has a larger area: a rectangle that is 7 inches by $\frac{3}{4}$ inch, or a square with side length of $2\frac{1}{2}$ inches? Show your reasoning.

Solution

The square is larger. Sample reasoning: Its area is $2\frac{1}{2} \times 2\frac{1}{2} = \frac{5}{2} \times \frac{5}{2}$, which is $\frac{25}{4}$ or $6\frac{1}{4}$ square inches. The rectangle has an area of $5\frac{1}{4}$ square inches because $7 \times \frac{3}{4} = \frac{21}{4}$.

