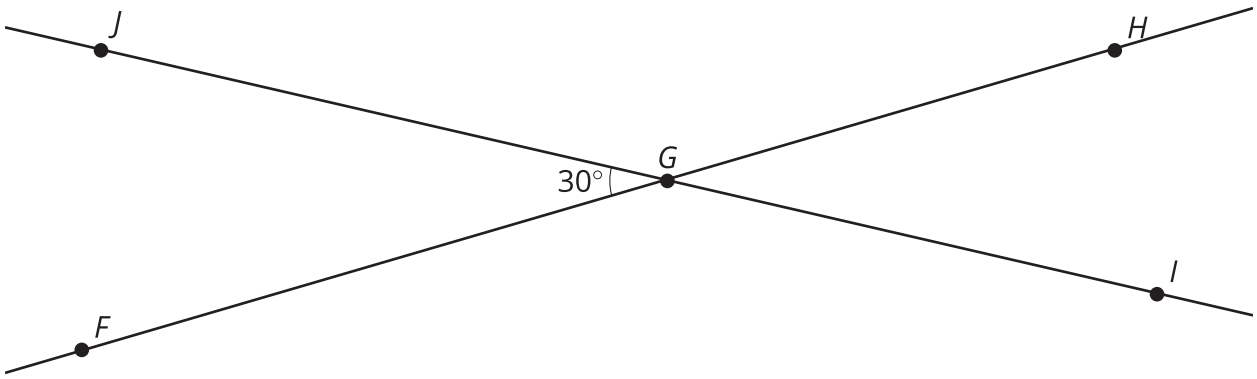
Alternate Interior Angles

Let's explore why some angles are always equal.

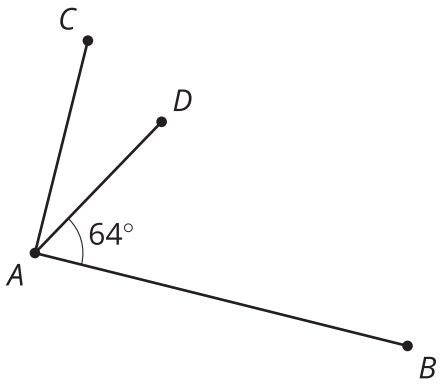
12.1 Angle Pairs

1. Find the measure of angle JGH. Explain or show your reasoning.



2. Find and label a second 30° angle in the diagram. Find and label an angle congruent to angle JGH.

3. Angle BAC is a right angle. Find the measure of angle CAD.



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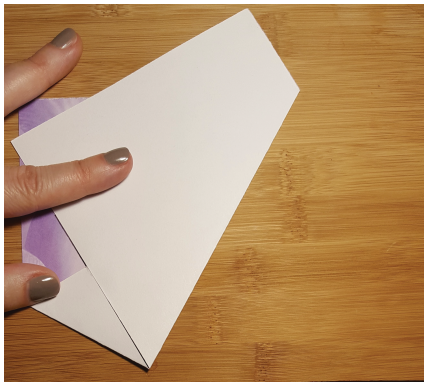
Are you ready for more?

Clare started with a rectangular piece of paper. She folded up one corner, and then folded up the other corner, as shown in the photos.



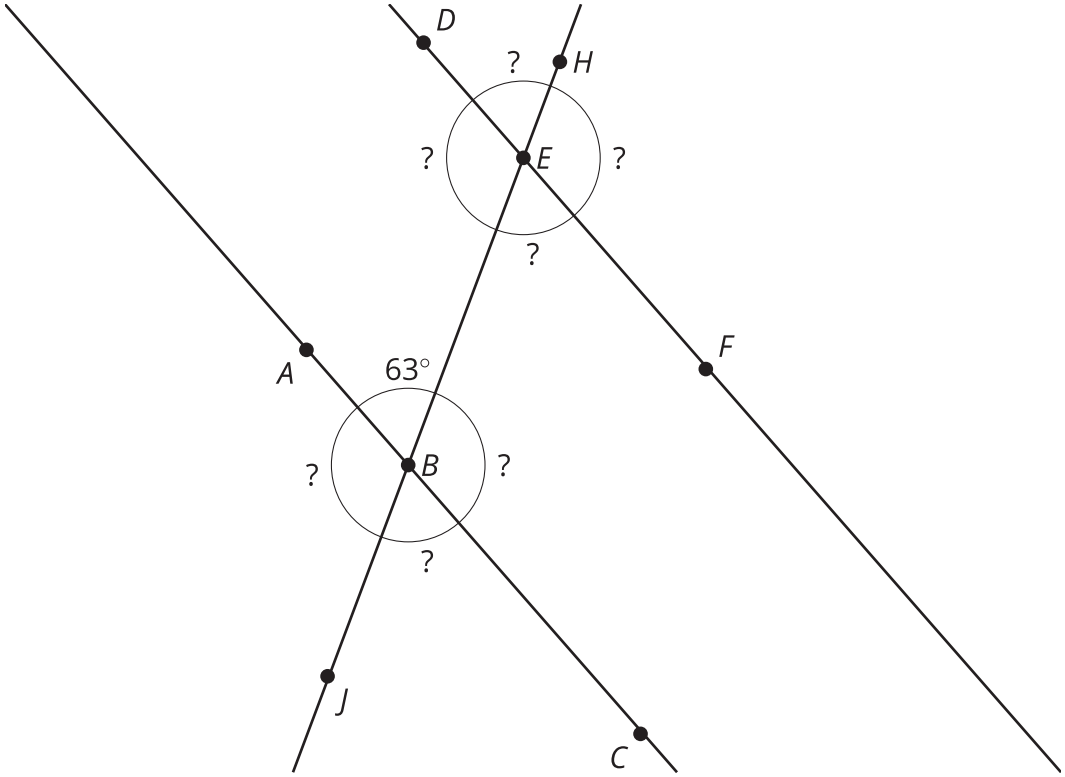




1. Try this yourself with any rectangular paper. Fold the left corner up at any angle, and then fold the right corner up so that the edges of the paper meet.
2. Clare thought that the angle at the bottom looked like a 90 degree angle. Does yours also look like it is 90 degrees?
3. Can you explain why the bottom angle *always has to be* 90 degrees? Hint: The third photo shows Clare's paper, unfolded. The crease marks are shown as dashed lines, and the line where the two paper edges met is shown as a solid line. Mark these on your own paper as well.

12.2 Cutting Parallel Lines with a Transversal

Lines AC and DF are parallel. They are cut by **transversal** HJ.



1. With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning.

2. What do you notice about the angles with vertex B and the angles with vertex E?

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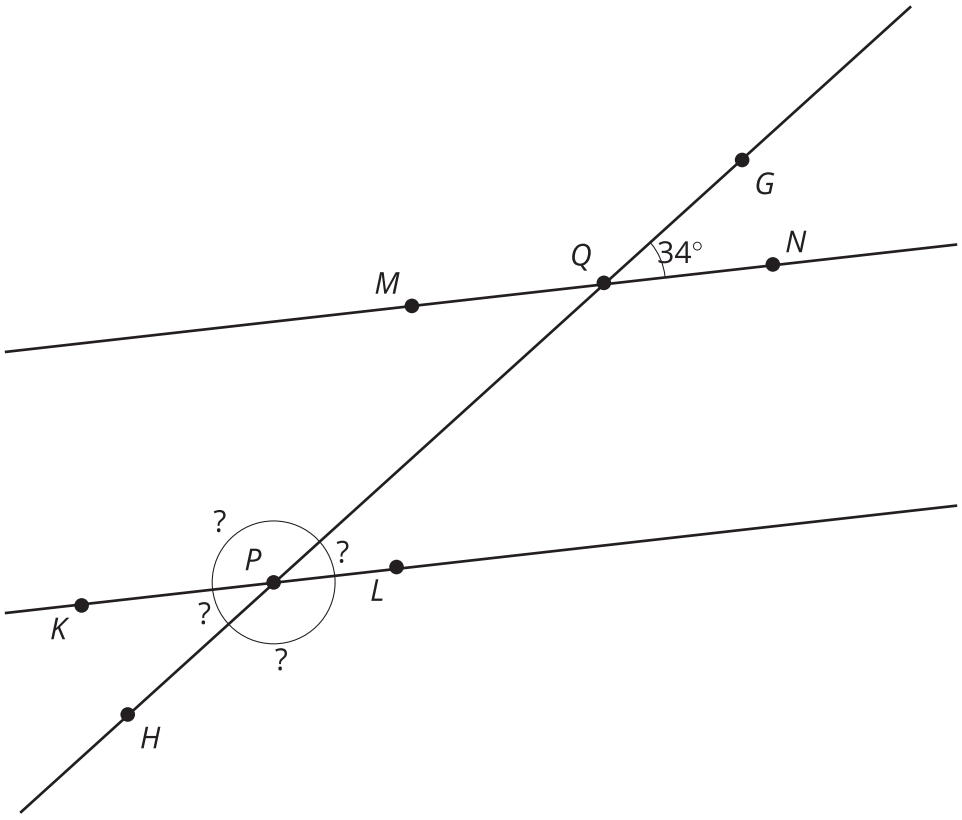
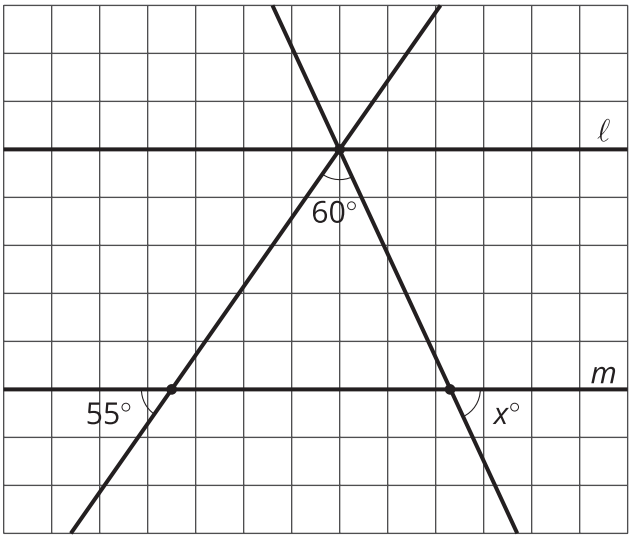
3. Using what you noticed, find the measures of the four angles at point P in the diagram. Lines KL and MN are parallel.



Are you ready for more?

Parallel lines ℓ and m are cut by two transversals that intersect ℓ at the same point. Two angles are marked in the figure. Find the measure x of the third angle.



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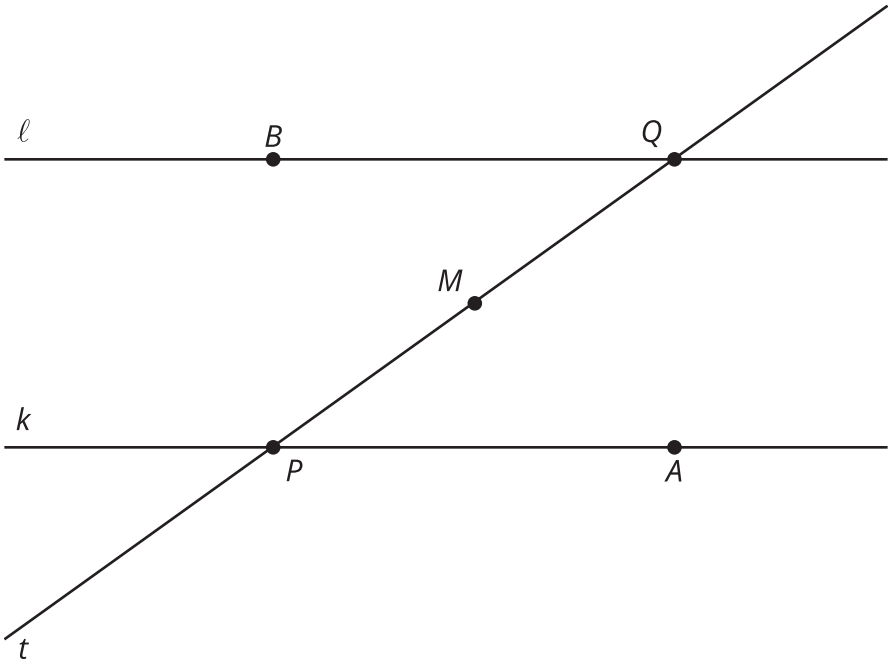
12.3 Alternate Interior Angles Are Congruent

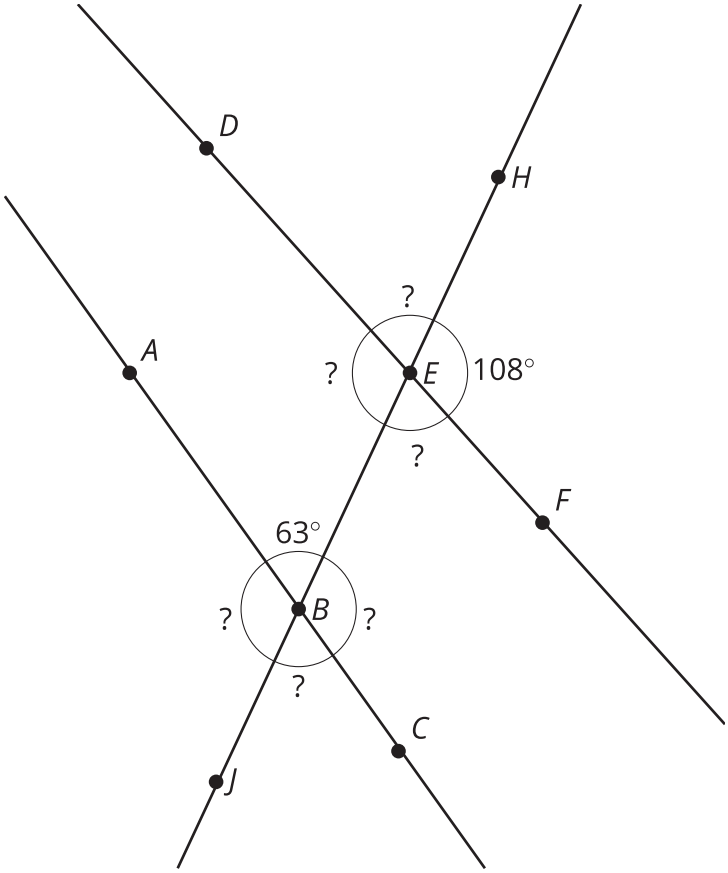


Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.

Find a rigid transformation showing that angles MPA and MQB are congruent.

12.4 Not Parallel

1. Lines DF and AC are not parallel in this image.

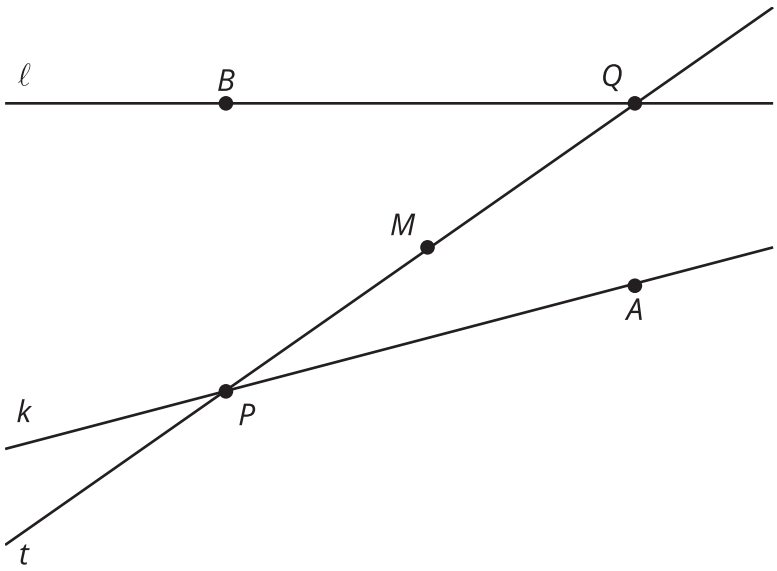


Find the missing angle measures around point E and point B.

What do you notice about the angles in this diagram?

2. Point M is the midpoint of line segment QP.

Can you find a rigid transformation that shows angle BQM is congruent to angle MPA? Explain your reasoning.



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12.5 Info Gap: Angle Finding

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. "Can you tell me _____?"
3. Explain to your partner how you are using the information to solve the problem. "I need to know _____ because . . . ."

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

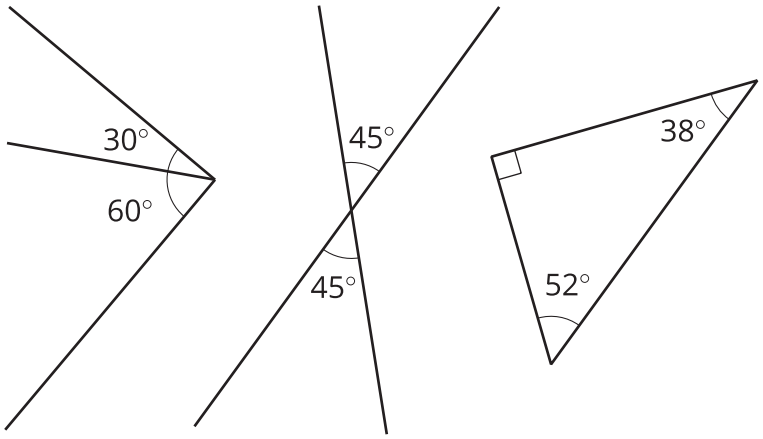
If your teacher gives you the data card:

1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, "Why do you need to know _____?"
3. Listen to your partner's reasoning and ask clarifying questions. Give only information that is on your card. Do not figure out anything for your partner!

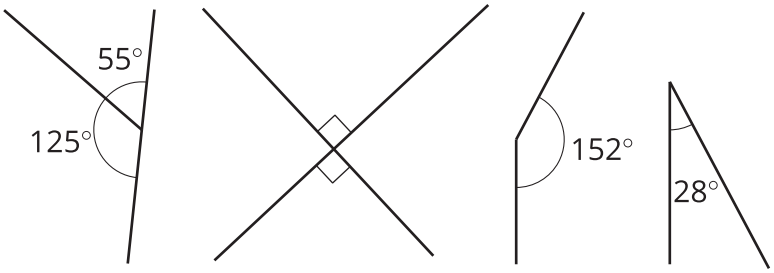
   These steps may be repeated.
4. Once your partner says there is enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Lesson 12 Summary

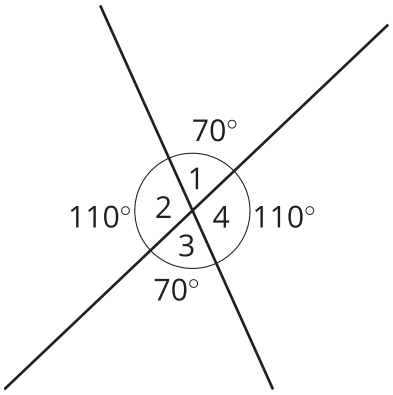
If two angle measures add up to 90°, then we say the angles are **complementary**. Here are three examples of pairs of complementary angles.



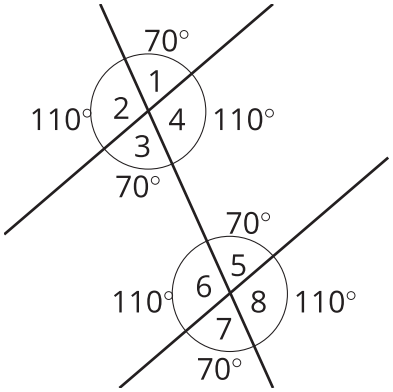
If two angle measures add up to 180°, then we say the angles are **supplementary**. Here are three examples of pairs of supplementary angles.



When two lines intersect, vertical angles are congruent, and adjacent angles are supplementary, so their measures sum to 180. For example, in this figure angles 1 and 3 are congruent, angles 2 and 4 are congruent, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.



When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.



Alternate interior angles are equal because a 180° rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections. Can you see how rotating 180° about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is 70° we use vertical angles to see that angle 3 is 70°, then we use alternate interior angles to see that angle 5 is 70°, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is 110° since $180 - 70 = 110$. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure 70°, and angles 2, 4, 6, and 8 measure 110°.