Lesson 7: Angle-Side-Angle Triangle Congruence

- Let’s see if we can prove other sets of measurements that guarantee triangles are congruent, and apply those theorems.

7.1: Notice and Wonder: Assertion

Assertion: Through 2 distinct points passes a unique line. Two lines are said to be *distinct* if there is at least 1 point that belongs to one but not the other. Otherwise, we say the lines are the same. Lines that have no point in common are said to be *parallel*.

Therefore, we can conclude: given 2 distinct lines, either they are parallel, or they have exactly 1 point in common.

What do you notice? What do you wonder?

7.2: Proving the Angle-Side-Angle Triangle Congruence Theorem

1. Two triangles have 2 pairs of corresponding angles congruent, and the corresponding sides between those angles are congruent. Sketch 2 triangles that fit this description.

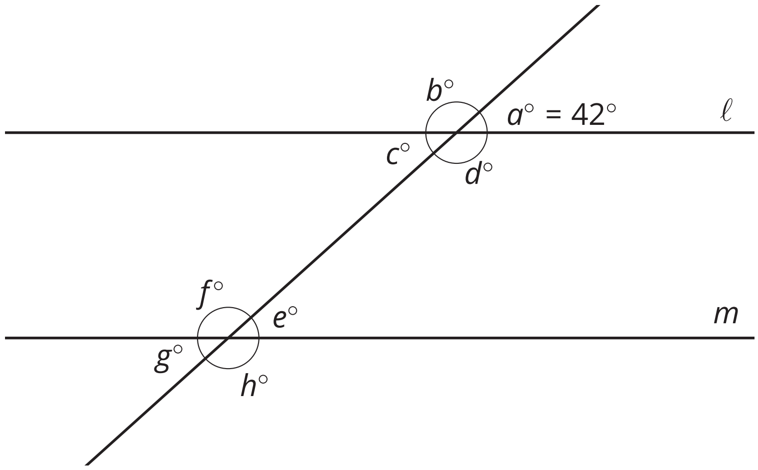
2. Label the triangles WXY and DEF, so that angle W is congruent to angle D, angle X is congruent to angle E, and side WX is congruent to side DE.

3. Use a sequence of rigid motions to take triangle WXY onto triangle DEF. For each step, explain how you know that one or more vertices will line up.

7.3: Find the Missing Angle Measures

Lines ℓ and m are parallel. $a = 42$. Find b, c, d, e, f, g, and h.

$\ell \parallel m$



7.4: What Do We Know For Sure About Parallelograms?

Quadrilateral $ABCD$ is a **parallelogram**. By definition, that means that segment AB is parallel to segment CD, and segment BC is parallel to segment AD.

1. Sketch parallelogram $ABCD$ and then draw an auxiliary line to show how $ABCD$ can be decomposed into 2 triangles.

2. Prove that the 2 triangles you created are congruent, and explain why that shows one pair of opposite sides of a parallelogram must be congruent.

Are you ready for more?

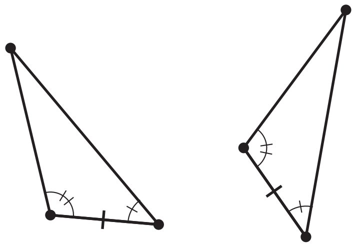
When we have 3 consecutive vertices of a polygon A, B, and C so that the triangle ABC lies entirely inside the polygon, we call B an *ear* of the polygon.

1. How many ears does a parallelogram have?
2. Draw a quadrilateral that has fewer ears than a parallelogram.
3. In 1975, Gary Meisters proved that every polygon has at least 2 ears. Draw a hexagon with only 2 ears.

Lesson 7 Summary

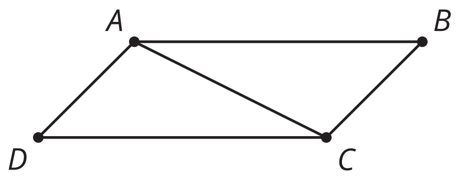
We know that in 2 triangles, if 2 pairs of corresponding sides and the pair of corresponding angles between the sides are congruent, then the triangles must be congruent. But we don't always know that 2 pairs of corresponding sides are congruent. For example, when proving that opposite sides are congruent in any parallelogram, we only have information about 1 pair of corresponding sides. That is why we need other ways than the Side-Angle-Side Triangle Congruence Theorem to prove triangles are congruent.

In 2 triangles, if 2 pairs of corresponding angles and the pair of corresponding sides between the angles are congruent, then the triangles must be congruent. This is called the *Angle-Side-Angle Triangle Congruence Theorem*.



When proving that 2 triangles are congruent, look at the diagram and given information and think about whether it will be easier to find 2 pairs of corresponding angles that are congruent or 2 pairs of corresponding sides that are congruent. Then check if there is enough information to use the Angle-Side-Angle Triangle Congruence Theorem or the Side-Angle-Side Triangle Congruence Theorem.

The Angle-Side-Angle Triangle Congruence Theorem can be used to prove that, in a **parallelogram**, opposite sides are congruent. A parallelogram is defined to be a quadrilateral with 2 pairs of opposite sides parallel.



We could prove that triangles ABC and CDA are congruent by the Angle-Side-Angle Triangle Congruence Theorem. Then we can say segment AD is congruent to segment CB because they are corresponding parts of congruent triangles.