Unit 3, Lesson 8

Polynomial Identities (Part 1)

Let's learn about polynomial identities.

8.1 Math Talk: Differences

Find the value of each expression mentally.

- $5^2 - 4^2$
- $6^2 - 5^2$
- $10^2 - 9^2$
- $41^2 - 40^2$

8.2 A Closer Look at Differences

1. Clare thinks the difference between the squares of two consecutive integers will always be the sum of the two integers. Is she right? Explain or show your reasoning.

   Pause here for a class discussion.

2. Andre thinks the difference between the squares of two consecutive even integers will always be 4 times the sum of the two integers. Is he right? Explain or show your reasoning.

Are you ready for more?

Noah says that the difference of two cubes is always divisible by the difference of the two numbers. Do you agree with Noah?

8.3 That Expression Is How Big?

Apply the distributive property to rewrite the following expressions without parentheses, combining like terms where possible. What do you notice?

1. $(x-1)(x+1)$

2. $(x-1)(x^2+x+1)$

3. $(x-1)(x^3+x^2+x+1)$

4. $(x-1)(x^4+x^3+x^2+x+1)$

5. $(x-1)(x^{20}+x^{19}+\ldots+x+1)$

Lesson 8 Summary

In earlier grades we learned how to do things like apply the distributive property and combine like terms to rewrite expressions in different ways. For example, $(2x+1)(x-3) = 2x^2 - 5x - 3$. The new algebraic expression on the right comes from writing the original on the left in a different way. More precisely, the expression on the left has the same value for all possible inputs x as the expression on the right, making them equivalent expressions. This is an example of an **identity**.

Two examples of identities seen in earlier grades are:

$$a^2 - b^2 = (a+b)(a-b)$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

For all possible values of a and b, the left and right sides of these equations are equal. In fact, the first of these identities can be extended to show that for any positive integer value of n the expression

$$(x-1)(x^{n-1} + x^{n-2} + \ldots + x^2 + x + 1)$$

is equivalent to

$$(x^n - 1)$$