Unit 1, Lesson 12

Defining Translations

Goals

- Comprehend that the term “translation” (in written and spoken language) requires specifying a directed line segment.
- Determine whether a figure is a translation of another.
- Draw translations of figures.

Learning Targets

- I can describe a translation by stating the directed line segment.
- I can draw translations.

Lesson Narrative

In this lesson, students refine their definition of translation. While they are practicing performing translations they are also formulating conjectures and developing their justification skills. These activities lead to the necessity of establishing the Parallel Postulate. In this lesson, students also encounter their first theorem of the course. **Theorems** are reserved for conjectures that students prove.

The concept of a **directed line segment**, a line segment with an arrow at one end specifying a direction, is introduced to give students language for efficiently describing the direction and length of a translation. A **translation** takes a point to another point so that the directed line segment from the original point to the image is parallel to the given directed line segment and has the same length and direction. Students know the term “line segment,” so the phrase “directed line segment” builds on a concept they already know and connects it to the concept of translations. The word “vector” is purposely avoided because the geometric interpretation of a vector should arise as a consequence of future work with vectors, not as a definition.

Students make arguments and critique the reasoning of others when they explain why translating a line segment results in a parallelogram (MP3).

Technology isn't required for this lesson, but there are opportunities for students to choose to use appropriate technology to solve problems. We recommend making technology available.

Standards

Building On	**8.G.A.1, 8.G.A.3**
Addressing	**HSG-CO.A.4**
Building Toward	**HSG-CO.A.4, HSG-CO.C**

Instructional Routines

- Draw It
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Materials to Gather

- Geometry toolkits (HS): Activity 2

Required Preparation

Lesson:

Students will continue adding to their reference chart in this lesson. Be prepared to add to the class display. The Blank Reference Chart for students and a teacher copy of a completed version are available in the black line masters for the unit.

If there are multiple sections of this course in the same classroom, consider hiding entries on the class reference chart and revealing them at the appropriate time rather than making multiple displays.

Student Facing Learning Goals

Let's translate some figures.

12.1 Notice and Wonder: Two Triangles and an Arrow

5 min

Warm-up

Activity Narrative

The purpose of this *Warm-up* is to elicit the idea that a translation takes each point in the same direction by the same distance, which will be useful when students investigate translations throughout this lesson. While students may notice and wonder many things about these images, a directed line segment's relation to triangles is the important discussion point. This prompt gives students opportunities to see and make use of structure (MP7). The specific structure they might notice is that each point on one triangle is the same distance and direction from the corresponding point on the other triangle.

Standards

Building On	**8.G.A.1, 8.G.A.3**
Building Toward	**HSG-CO.A.4**

Instructional Routines

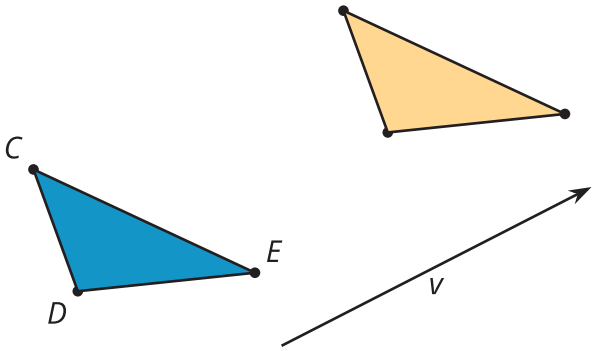
- Notice and Wonder

Launch

Display the image for all to see. Ask students to think of at least one thing they notice and at least one thing they wonder. Give students 1 minute of quiet think time and then 1 minute to discuss with their partner the things they notice and wonder, and follow with a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- There are two triangles.
- There are letters for some points but not all points.
- There is an arrow.
- Corresponding sides of the triangles appear to be parallel.

Things students may wonder:

- How does the arrow relate to the two triangles?
- Are the two triangles congruent?
- Does that arrow fit between all three pairs of corresponding points?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After all responses have been recorded without commentary or editing, ask students, "Is there anything on this list you are wondering about now?" Encourage students to respectfully disagree, ask for clarification, or point out contradicting information.

If connecting points of one triangle to their corresponding points on the other does not come up during the conversation, ask students to discuss this idea.

12.2 What's the Point: Translations

15 min

Activity Narrative

In this activity, students explore translations without a coordinate grid by identifying and describing transformations. Monitor for students who notice parallel lines formed by directed line segments or formed by points and their images.

Standards

Building On	**8.G.A.1, 8.G.A.3**
Addressing	**HSG-CO.A.4**
Building Toward	**HSG-CO.C**

Launch

Suggest that students either use tracing paper or two different colors to clearly differentiate the two transformations.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example:

- "I noticed _____, so I _____."
- "_____ could/couldn't be true because _____."

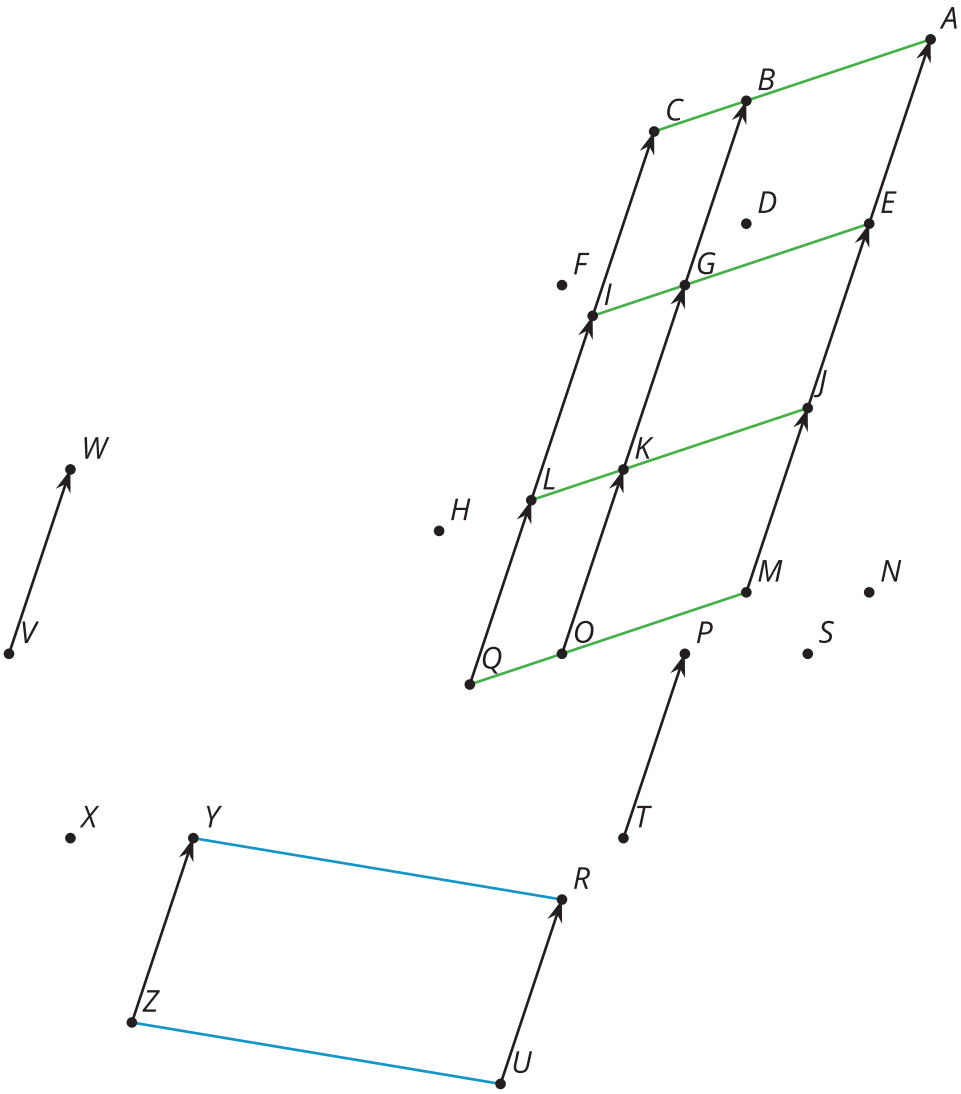
Supports accessibility for: Language, Organization

Student Task Statement

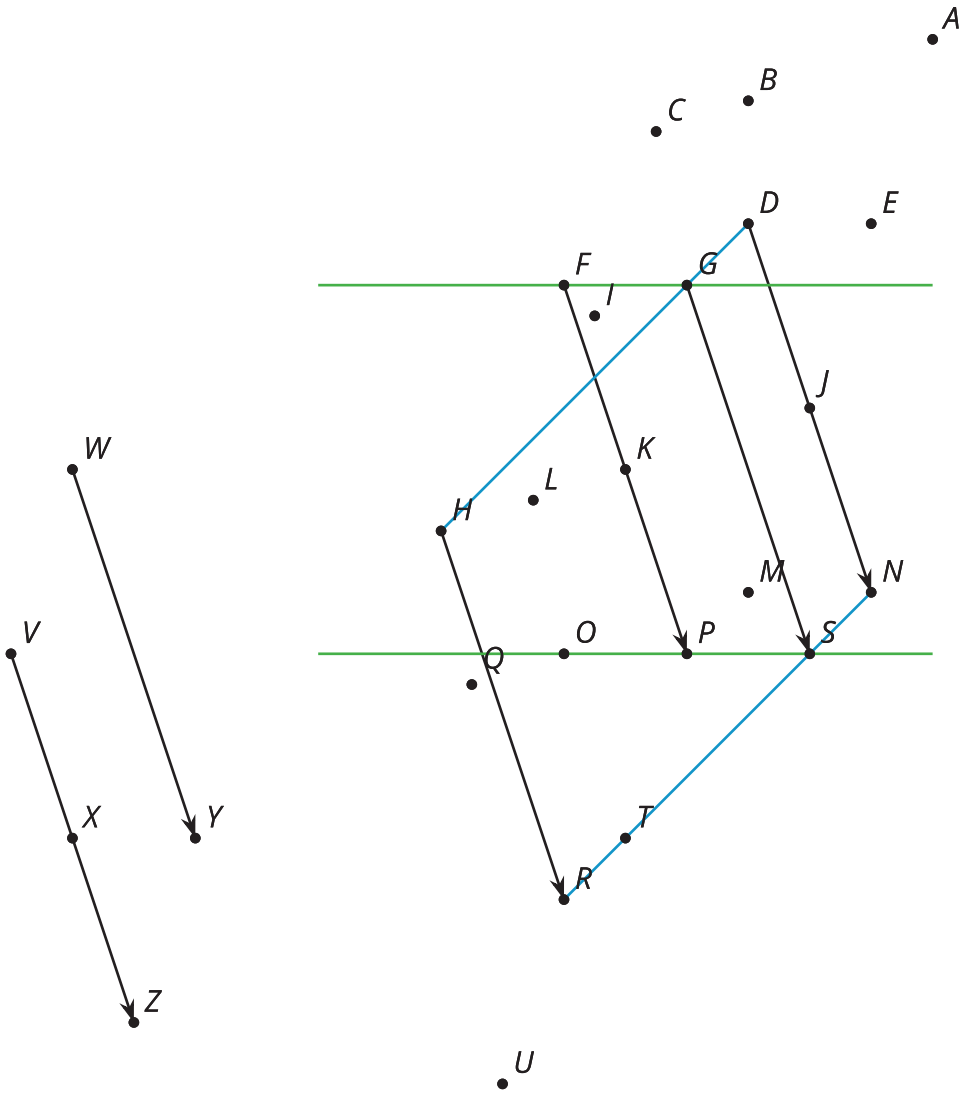
1. After a translation, the image of V is W. Find at least 3 other points that are taken to a labeled point by this same translation.
2. Write at least 1 conjecture about translations.
3. In a new translation, the image of V is Z. Find at least 3 other points that are taken to a labeled point by the new translation.
4. Are your conjectures still true for the new translation?

Student Response

1. See image:



2. Sample response: Translations take lines to parallel lines. The arrows make parallel lines.
3. See image:



4. Answers vary.

Building on Student Thinking

Students may need to be reminded of the tools in their geometry toolkits, such as tracing paper, straightedges, and compasses.

Activity Synthesis

Invite students to share what they conjectured.

Highlight for students that connecting each original point to each image results in arrows that are all the same length and going in the same direction. Tell students that we call these arrows **directed line segments**. In other words, a directed line segment is a line segment with a direction to it. A directed line segment conveys the direction and distance that each point is translated.

If no student conjectures about translation taking lines to parallel lines, display the images of student solutions with lines drawn in. There will be time in subsequent activities to explore this idea further. Students are only conjecturing at this point.

12.3 Translating Triangles

15 min

Activity Narrative

There is a digital version of this activity.

This activity highlights that translations take lines to parallel lines and segments to segments of the same length. Both of these properties will be used in future lessons to prove theorems.

Monitor for different ways students justify their claims about parallel lines and equal distances. It is not expected that students come up with rigorous, formal arguments at this point. It is important to encourage students to justify their ideas to begin the transition to more formal arguments.

Making dynamic geometry software available gives students an opportunity to choose appropriate tools strategically (MP5).

Standards

Building On	**8.G.A.1, 8.G.A.3**
Addressing	**HSG-CO.A.4**
Building Toward	**HSG-CO.C**

Instructional Routines

- Draw It
- MLR8: Discussion Supports

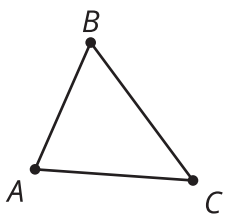
Launch

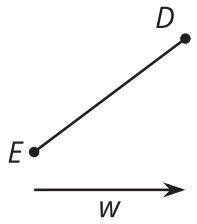
Arrange students in groups of 2. After quiet work time, ask students to compare their responses to their partner's and decide if they are both correct, even if they are different. Follow with whole-class discussion.

Access for English Language Learners

MLR8 Discussion Supports. Display sentence frames to support students in producing statements about the properties of the translated figures: "BC and $B'C'$ are ____ because ____" or "Shape $EDD'E'$ is a ____ because ____."
Advances: Writing, Conversing

Student Task Statement





1. Translate triangle ABC by the **directed line segment** from A to C.

a. What is the relationship between line BC and line $B'C'$? Explain your reasoning.

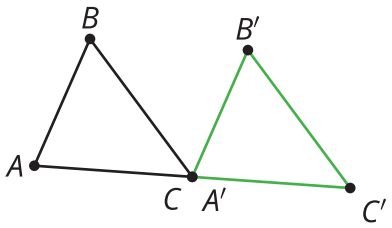
b. How does the length of segment BC compare to the length of segment $B'C'$? Explain your reasoning.

2. Translate segment DE by directed line segment w. Label the new endpoints D' and E'.

a. Connect D to D' and E to E'.

b. What kind of shape did you draw? What properties does it have? Explain your reasoning.

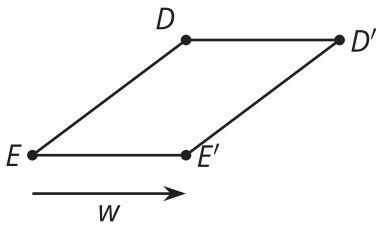
Student Response



1.

a. Lines BC and $B'C'$ are parallel because the translation takes each point on segment BC the same distance in the same direction.

b. Segments BC and $B'C'$ have the same length because a translation is a rigid transformation.



2.

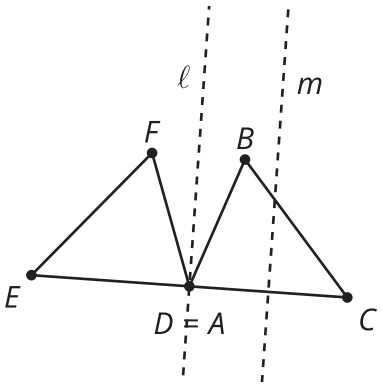
a. See image.

b. Sample responses: The shape looks like a parallelogram. Segment EE' is parallel to segment DD' because they are both parallel to directed line segment w. Segment EE' has the same length as segment DD' because they are both the same length as directed line segment w.

Are You Ready for More?

1. On triangle ABC in the task, use a straightedge and compass to construct the line which passes through A and is perpendicular to AC. Label it ℓ. Then construct the perpendicular bisector of AC and label it m. Draw the reflection of ABC across the line ℓ. Since the label $A'B'C'$ is used already, label it DEF instead.
2. What is the reflection of DEF across the line m?
3. Explain why this is cool. What does it tell you about translations?

Extension Student Response

1. 

2. It is the same $A'B'C'$ that was already drawn.
3. Sample response: A translation can be done as two reflections across lines perpendicular to the directed line segment where the distance between them is half the length of the segment!

Activity Synthesis

The purpose of this discussion is to highlight the fact that translations take lines to parallel lines and segments to segments of equal length. Here are some questions for discussion:

- "How do you know that lines BC and $B'C'$ are parallel?" (Lines BC and $B'C'$ are parallel because the translation took each point on segment BC the same distance in the same direction. One way to think of what makes lines parallel is that all pairs of corresponding points are the same distance apart.)
- "How do you know that segments BC and $B'C'$ are the same length?" (Translations are rigid transformations that take segments to segments of equal length.)

Lesson Synthesis

Explain to students that there are two facts related to translations and parallel lines that will come up several times in future lessons and units:

- Given a line and a point off the line, there is a unique parallel line that goes through the point.
- Translations take lines to parallel lines or to themselves.

Display a line ℓ and a point not on the line, B. Ask students, "What are the possible lines through B? How many of them are parallel to ℓ?" (Infinite lines go through B, but only one is parallel to ℓ.) Tell students the idea that there is one unique line parallel to ℓ that goes through B is called the *Parallel Postulate*. It's an observation that seems to be true, but there is no way to prove or disprove it. We will take it as an assertion.

Explain to students that translations don't make sense without the Parallel Postulate because the definition of translating a point A by a directed line segment t assumes there is only one line through A that is parallel to t.

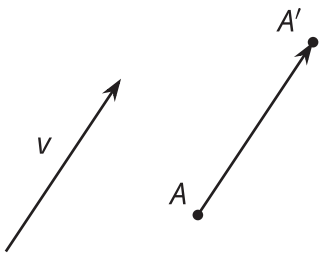
Add the following definition, assertion, and theorem to the class reference chart, and ask students to add them to their reference charts.

Translation is a rigid transformation that takes a point to another point so that the directed line segment from the original point to the image is parallel to the given line segment and has the same length and direction.

"Translate _(object)_ by the directed line segment _(name or from [point] to [point])_."

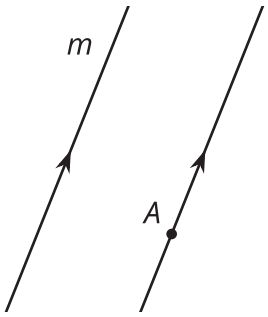
(Definition)

Translate A by the directed line segment v.



Parallel Postulate: Given a line m and a point A that is not on m, there is exactly one line that goes through A that is parallel to m.

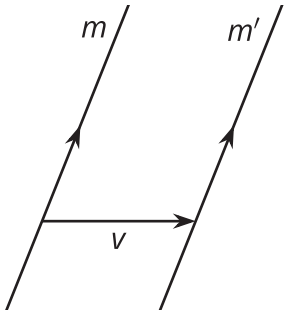
(Assertion)



Translations take lines to parallel lines or to themselves.

(Theorem)

$m \parallel m'$



12.4 What Went Wrong? Translation

Cool-down

5 min

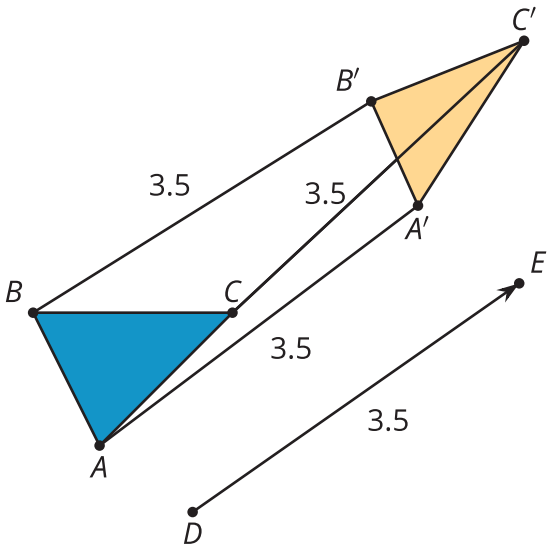
Standards

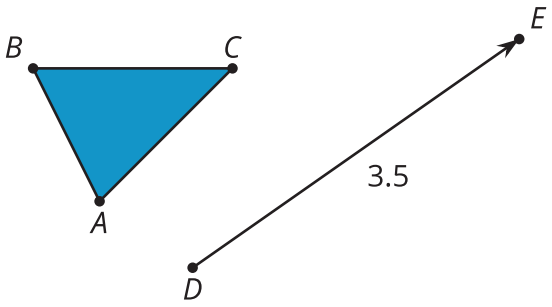
Addressing **HSG-CO.A.4**

Student Task Statement

Priya tried to translate triangle ABC by the directed line segment from D to E. She knows something went wrong because the image isn't congruent to the original figure.

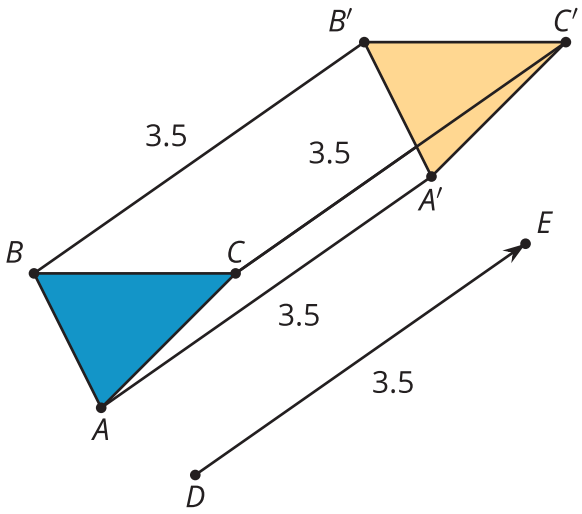
1. What is one idea that Priya probably understands about translations?
2. What is one idea that Priya doesn't understand about translations?
3. Translate triangle ABC by the directed line segment from D to E.





Student Response

1. Priya knows that translations take points to a new point the same distance away.
2. Priya doesn't understand that the movements have to be in the same direction and that all the segments should be parallel.



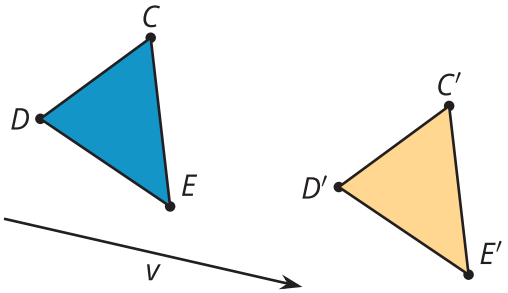
3.

Responding to Student Thinking

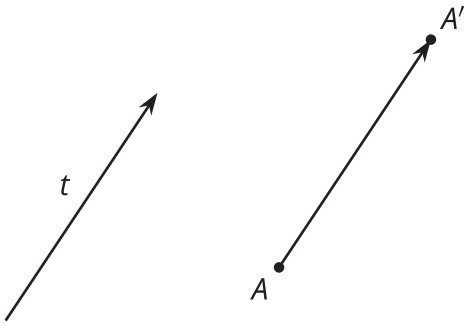
More Chances
Students will have more opportunities to understand the mathematical ideas addressed here. There is no need to slow down or add additional work to the next lessons.

Lesson 12 Summary

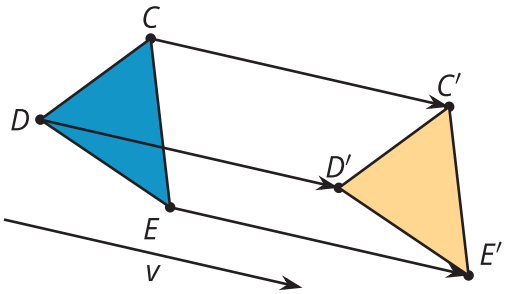
A translation slides a figure a given distance in a given direction with no rotation. The distance and direction are given by a **directed line segment**. The arrow of the directed line segment specifies the direction of the translation, and the length of the directed line segment specifies how far the figure gets translated.



More precisely, a **translation** of a point A by a directed line segment t is a transformation that takes A to A' so that the directed line segment AA' is parallel to t, goes in the same direction as t, and is the same length as t.



Here is a translation of 3 points. Notice that the directed line segments CC', DD', and EE' are each parallel to v, go in the same direction as v, and are the same length as v.



Also notice that segment CD is parallel to segment $C'D'$. We proved that this would always be true, so we can write a theorem that says translations take lines to parallel lines or to themselves. A **theorem** is a statement that has been proved mathematically.

Glossary

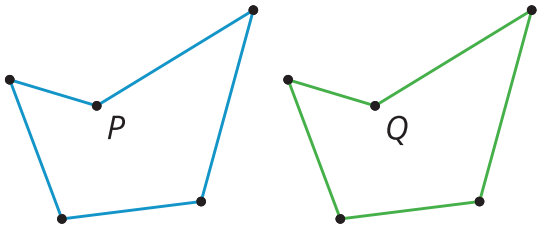
- directed line segment
- theorem
- translation

Lesson 12 Practice Problems

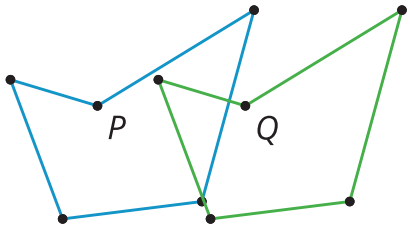
1 Student Task Statement

Match each directed line segment with the translation from Polygon P to Polygon Q by that directed line segment.

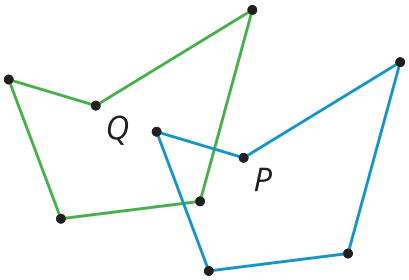
Translation 1



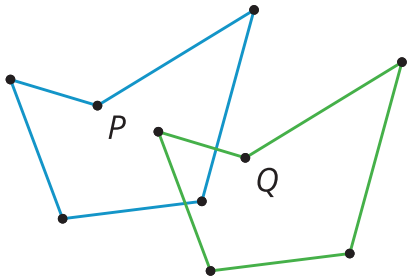
Translation 2



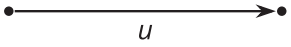
Translation 3



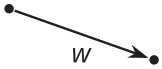
Translation 4



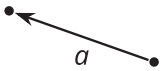
A.



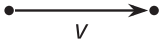
B.



C.



D.



1. Translation 1
2. Translation 2
3. Translation 3
4. Translation 4

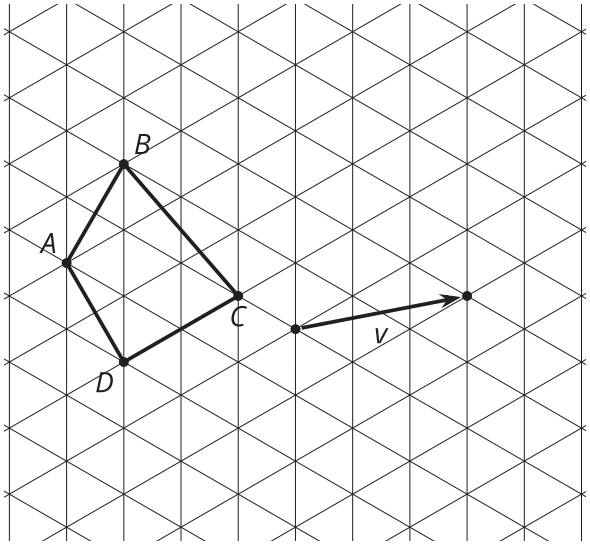
Solution

- A matches 1

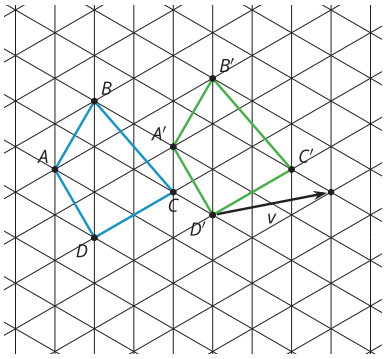
- B matches 4
- C matches 3
- D matches 2

2 Student Task Statement

Draw the image of quadrilateral $ABCD$ when translated by the directed line segment v. Label the image of A as A', the image of B as B', the image of C as C', and the image of D as D'.



Solution



3 Student Task Statement

Which statement is true about a translation?

A. A translation takes a line to a parallel line or to itself.

B. A translation takes a line to a perpendicular line.

C. A translation requires a center of translation.

D. A translation requires a line of translation.

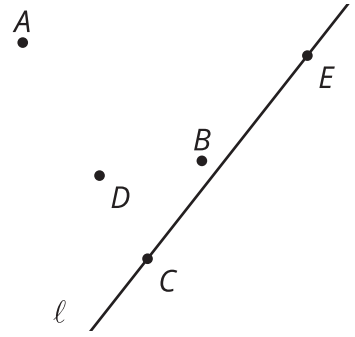
Solution

A

4 from Unit 1, Lesson 11

Student Task Statement

Select **all** the points that stay in the same location after being reflected across line ℓ.



A. A

B. B

C. C

D. D

E. E

Solution

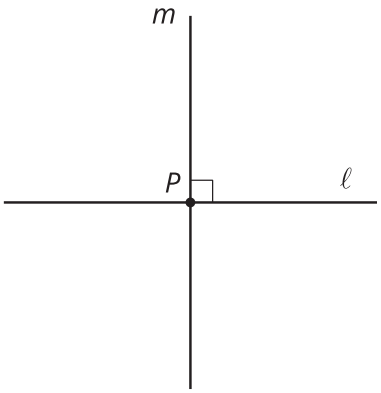
C, E

5 from Unit 1, Lesson 11

Student Task Statement

Lines ℓ and m are perpendicular. A point Q has this property: Rotating Q 180 degrees using center P has the same effect as reflecting Q over line m.

$m \perp \ell$



a. Give two possible locations of Q.

b. Do all points in the plane have this property?

Solution

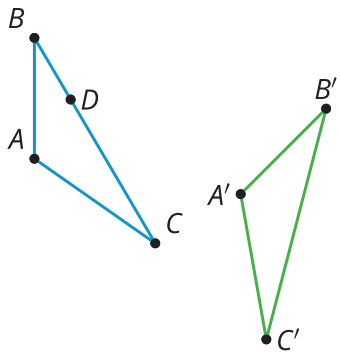
a. Sample response: any two points on line ℓ

b. No, only points on ℓ.

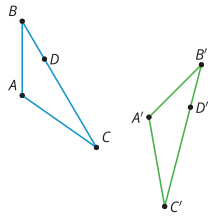
6 from Unit 1, Lesson 10

Student Task Statement

There is a sequence of rigid transformations that takes A to A', B to B', and C to C'. The same sequence takes D to D'. Draw and label D':



Solution



7 from Unit 1, Lesson 6

Student Task Statement

Two distinct lines ℓ and m are each perpendicular to the same line n.

a. What is the measure of the angle where line ℓ meets line n?

b. What is the measure of the angle where line m meets line n?

Solution

a. 90 degrees

b. 90 degrees