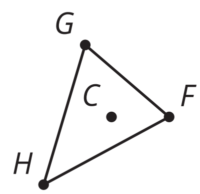
Unit 3, Lesson 3

Measuring Dilations

Let’s dilate polygons.

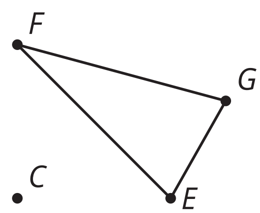
3.1 Dilating Out

Dilate triangle FGH using center C and a scale factor of 3.



3.2 All the Scale Factors

Here is a center of dilation and a triangle.



1. Measure the sides of triangle EFG (to the nearest mm).

2. Your teacher will assign you a scale factor. Predict the relative lengths of the original figure and the image after you dilate by your scale factor.

3. Dilate triangle EFG using center C and your scale factor.
4. How does your prediction compare to the image you drew?

5. Use tracing paper to copy point C, triangle EFG, and your dilation. Label your tracing paper with your scale factor.
6. Align your tracing paper with your partner's. What do you notice?

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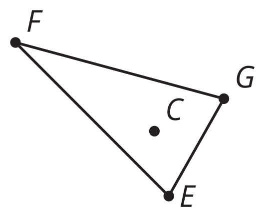
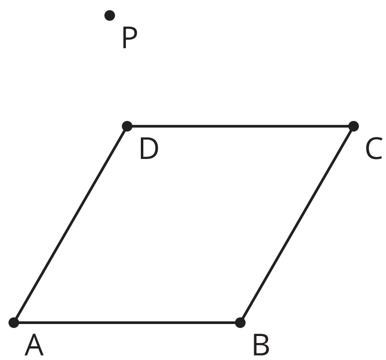
Are you ready for more?



1. Dilate triangle EFG using center C and scale factors:
   a. $\frac{1}{2}$
   b. 2
2. What scale factors would cause some part of triangle $E'F'G'$ to intersect some part of triangle EFG?

3.3 What Stays the Same?

1. Dilate quadrilateral $ABCD$ using center P and your scale factor.



2. Complete the table.

Ratio	$\frac{PA'}{PA}$	$\frac{PB'}{PB}$	$\frac{PC'}{PC}$	$\frac{PD'}{PD}$
Value				

3. What do you notice? Can you prove your conjecture?

4. Complete the table.

Ratio	$\frac{B'A'}{BA}$	$\frac{C'B'}{CB}$	$\frac{D'C'}{DC}$	$\frac{A'D'}{AD}$
Value				

5. What do you notice? Does the same reasoning you just used also prove this conjecture?

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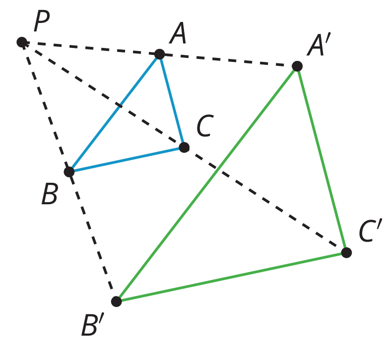
Lesson 3 Summary

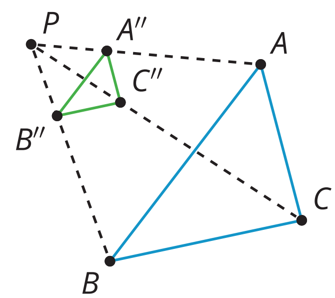
We know that a *dilation* with center P and positive *scale factor,* k, takes a point A along the ray PA to another point whose distance is k times farther away from P than A is.

The triangle $A'B'C'$ is a dilation of the triangle ABC with center P and with a scale factor of 2. So A' is 2 times farther away from P than A is, B' is 2 times farther away from P than B is, and C' is 2 times farther away from P than C is.



Because of the way dilations are defined, all of these quotients give the scale factor: $\frac{PA'}{PA} = \frac{PB'}{PB} = \frac{PC'}{PC} = 2.$

If triangle ABC is dilated from point P with scale factor $\frac{1}{3}$, then each vertex in $A''B''C''$ is on the ray from P through the corresponding vertex of ABC, and the distance from P to each vertex in $A''B''C''$ is one-third as far as the distance from P to the corresponding vertex in ABC.



$$\frac{PA''}{PA} = \frac{PB''}{PB} = \frac{PC''}{PC} = \frac{1}{3}$$

The dilation of a line segment is longer or shorter according to the same ratio given by the scale factor. In other words, if segment AB is dilated from point P with a scale factor of k, then the length of segment AB is multiplied by k to get the corresponding length of $A'B'$.

$$\frac{A''B''}{AB} = \frac{B''C''}{BC} = \frac{A''C''}{AC} = k.$$

Corresponding side lengths of the original figure and dilated image are all in the same proportion, and are related by the same scale factor, k.

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