Unit 3, Lesson 18

Using Equations to Solve for Unknown Angles

Goals

- Determine whether a given equation represents the relationship between angles shown in a diagram.
- Solve an equation that represents a relationship between angle measures, and explain (in writing and using other representations) the reasoning.
- Write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent the relationship between angles in a given diagram.

Learning Targets

- I can write an equation to represent a relationship between angle measures and solve the equation to find unknown angle measures.

Lesson Narrative

In this lesson, students practice writing and solving equations of the form $px + q = r$ in the context of finding unknown angle measures. Students examine the structure of angle relationships as they compare a diagram to possible equations that could be used to represent the angle relationships (MP7).

The angles used in this lesson connect to earlier work with supplementary, complementary, and vertical angles, giving additional opportunities to build fluency with these concepts.

Teacher Notes for IM 6–8 Math Accelerated v.360
Use this optional lesson any time after Lesson 10 of this unit.

This lesson includes a set of practice problems, even though the last lesson in a unit typically does not. The problems are included here to give students more practice writing and solving equations to find unknown angles. If time is limited, consider focusing on practice problems 1–3.

Standards

Building On	**7.EE.B.4**
Addressing	**7.EE.B.4, 7.G.B.5**

Instructional Routines

- MLR8: Discussion Supports

Student Facing Learning Goals

Let's figure out missing angles using equations.

18.1 Is This Enough?

Warm-up

5 min

Activity Narrative

In this activity, students consider whether there is enough information given to solve for the unknown angle measures. In previous lessons, students were given the measures of some angles in a figure and asked to solve for another. In this warm-up, the figure contains two unknowns and students are asked to critique Tyler's thinking (MP3).

The discussion addresses the case in which angles a and b are equal to each other, in preparation for future activities in this lesson that have multiple unknown angles with the same measure.

Standards

Building On **7.EE.B.4**

Addressing **7.G.B.5**

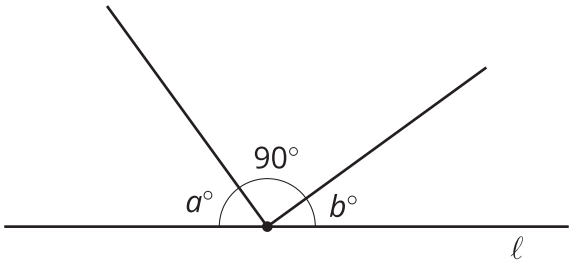
Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time, followed by time to discuss their reasoning with their partner. Follow with a whole-class discussion.

Student Task Statement

Tyler thinks that this figure has enough information to figure out the values of a and b.

Do you agree? Explain your reasoning.



Student Response

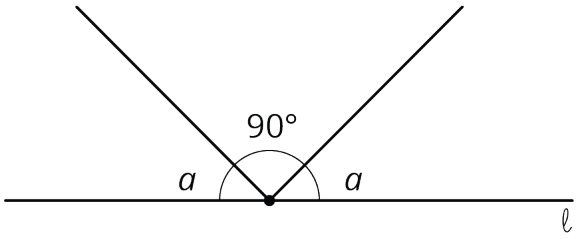
I disagree with Tyler. Sample reasoning: We don't know how much bigger a is than b. All we know for sure is that $a + b = 90$.

Activity Synthesis

Invite students to share their reasoning until they reach an agreement that Tyler is incorrect.

Ask students to come up with an equation to represent the angle measures in the figure. ($a + 90 + b = 180$, or equivalent) Record their answers for all to see.

Display this image. Invite students to share how this figure is the same as the figure from the task and how it is different.



If students do not mention any of these points, make sure to point them out:

- Some things that are the same are the facts that there are still two angles with unknown measures and the measures of the three angles add up to 180 degrees. The two unknown angles are still complementary.
- The main difference is that the two unknown angles have the same measure.
- This figure can be represented with the equation $a + 90 + a = 180$, or equivalent.
- Because both unknown angles have the same measure, we have enough information to know the value of a.
- $a = 45$

18.2 What Does It Look Like?

15 min

Activity Narrative

The purpose of this activity is for students to practice solving equations that represent relationships between angles, in preparation for the next activity where students will write such equations themselves. In this activity, they compare the diagrams to possible equations in order to make sense of the structure of the equations and the angle relationships that they represent (MP7).

Standards

Addressing **7.EE.B.4, 7.G.B.5**

Launch

Tell students that each diagram has two possible equations, and that their job is to choose the equation that best represents a relationship between angles in the diagram. Then they are to solve their chosen equation.

Keep students in the same groups. Give 5 minutes of quiet work time, followed by time to discuss reasoning with a partner. Follow with a whole-class discussion.

Tell students they can assume that the angles that appear to be right angles are 90 degrees. Students may also use an index card or corner of a piece of paper to identify any right angles, if needed.

Access for Students with Disabilities

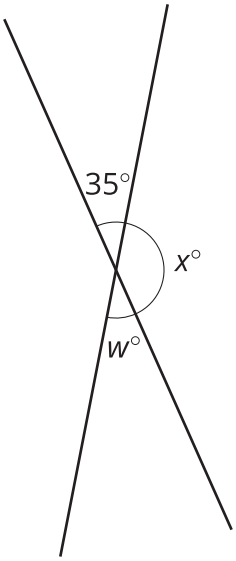
Engagement: Provide Access by Recruiting Interest. Provide choice. Invite students to start by labeling any angles they can find with the angle's degree measure.
Supports accessibility for: Organization, Attention

Student Task Statement

Elena and Diego each wrote equations to represent these diagrams. For each diagram, decide which equation you agree with, and then solve it.

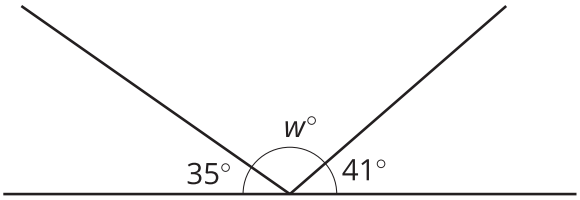
1. Elena: $x = 35$

   Diego: $x + 35 = 180$



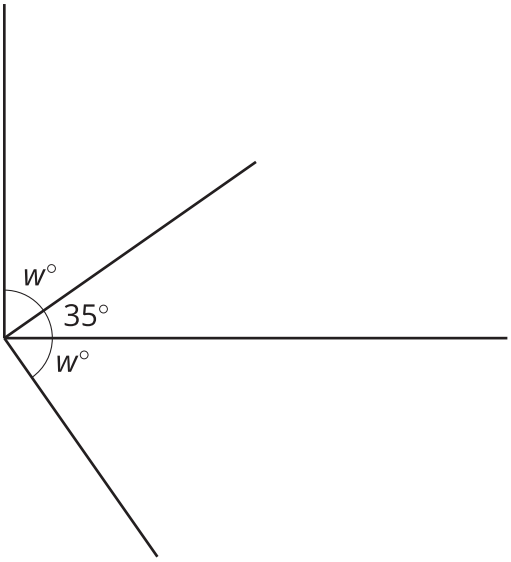
2. Elena: $35 + w + 41 = 180$

   Diego: $w + 35 = 180$



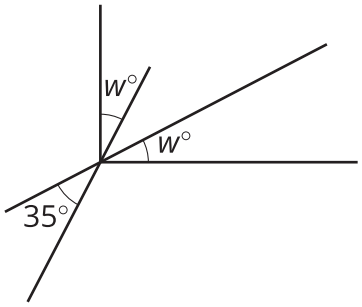
3. Elena: $w + 35 = 90$

   Diego: $2w + 35 = 90$



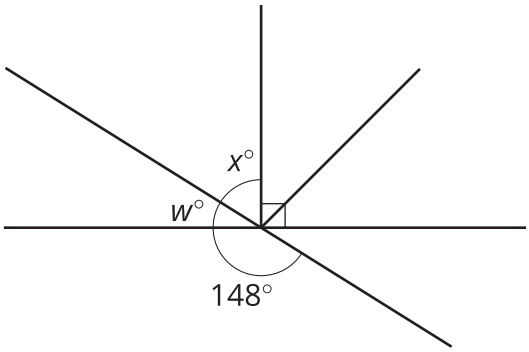
4. Elena: $2w + 35 = 90$

   Diego: $w + 35 = 90$



5. Elena: $w + 148 = 180$

   Diego: $x + 90 = 148$



Student Response

1. Diego's equation: $x + 35 = 180$. Solution: 145.
2. Elena's equation: $35 + w + 41 = 180$. Solution: 104.
3. Elena's equation: $w + 35 = 90$. Solution: 55.
4. Elena's equation: $2w + 35 = 90$. Solution: 27.5.
5. Both equations. $w = 32$ and $x = 58$.

Activity Synthesis

Select students to share equations they agreed with and angle measures they found for each problem. As students share their explanations, consider asking these questions:

- "Where do you see the relationship expressed in the equation in the given figure? (and vice versa)"
- "Did you and your partner agree on the equations and angle measures?"

For the last question, have students who used different equations to figure out the unknown angle measures share their explanations. Ask students:

- "What angle relationship did you need to recognize in order to use Elena's equation?" (That the angle with a measure of w degrees and the angle measuring 148 degrees were supplementary.)
- "What angle relationship did you need to recognize in order to use Diego's equation?" (That the angle measuring 148 degrees formed a vertical angle with the combined right angle and angle measuring x degrees.)
- "Does either method get us the same answer for both unknown angle measures?" (Yes.)

Explain to students that there might be multiple ways to get an answer because of the many angle relationships found in some figures. In the next activity, encourage them to look for different methods.

18.3 Calculate the Measure

10 min

Activity Narrative

In this activity, students come up with equations that represent the relationships between angles in a figure. Then, students solve their equation to find each unknown angle measure.

Standards

Addressing **7.EE.B.4, 7.G.B.5**

Instructional Routines

- MLR8: Discussion Supports

Launch

Give students 2–3 minutes of quiet work time, followed by a whole-class discussion.

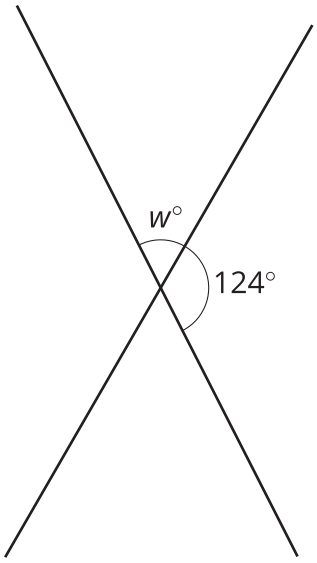
Monitor for students who represent their thinking using equations, and select these students to share during the whole-class discussion.

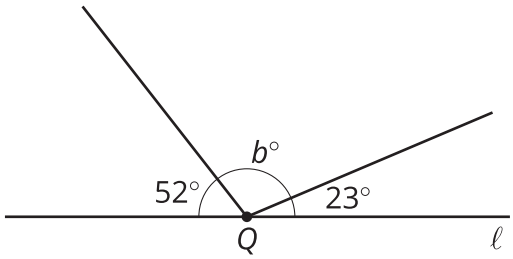
Access for Students with Disabilities

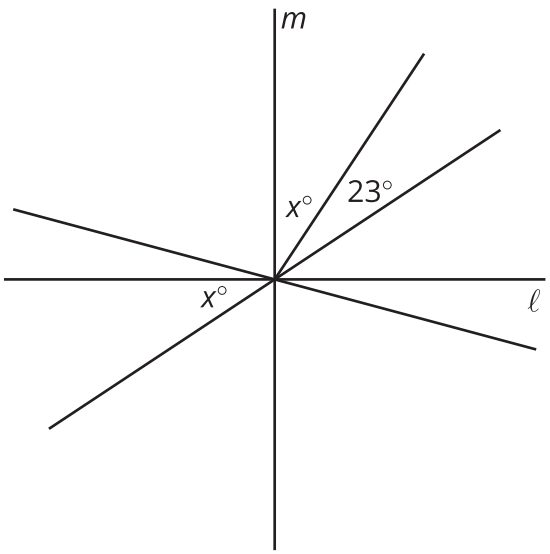
Representation: Internalize Comprehension. Activate or supply background knowledge by asking students to start by looking for any vertical, complementary, and supplementary angles. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

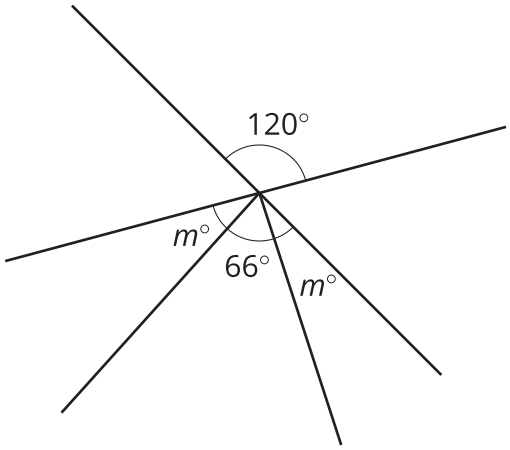
Student Task Statement

Find the unknown angle measures. Show your thinking. Organize it so it can be followed by others.









Student Response

- $w = 56$. Sample reasoning: $2(w + 124) = 360$, $w + 124 = 180$, $w = 180 - 124$
- $b = 105$. Sample reasoning: $b + 52 + 23 = 180$, $b = 180 - (52 + 23)$
- $x = 33.5$ or equivalent. Sample reasoning: $2x + 23 = 90$, $2x = 90 - 23$, $x = \frac{1}{2}(90 - 23)$
- $m = 27$. Sample reasoning: $2m + 66 = 120$, $2m = 120 - 66$, $m = \frac{1}{2}(120 - 66)$

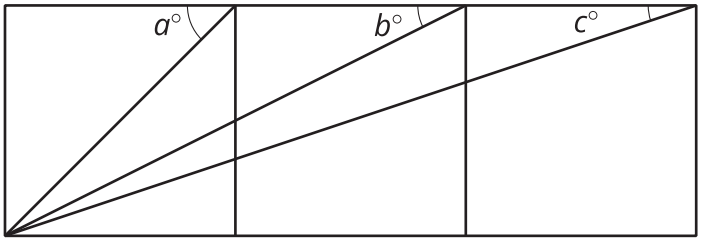
Building on Student Thinking

If students struggle to see the angle relationships in the figures, consider asking:

- "What types of angle relationships do you see?"
- "How can those angle relationships help you find the unknown angle measure?"

Are You Ready for More?

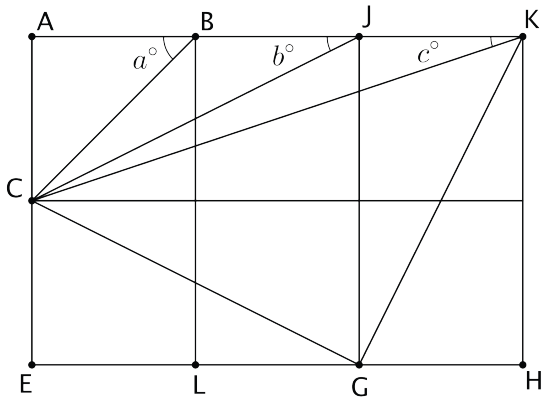
The diagram contains three squares. Three additional segments have been drawn that connect corners of the squares. We want to find the exact value of $a + b + c$.



1. Use a protractor to measure the three angles. Use your measurements to conjecture about the value of $a + b + c$.
2. Find the exact value of $a + b + c$ by reasoning about the diagram.

Extension Student Response

$a + b + c = 90$. Measuring carefully with a protractor is convincing, but there are many ways to show that $a + b + c$ is exactly 90°. One way is to expand the diagram with more squares and draw some more segments. Look at the three adjacent angles with vertices at point K. The measure of angle GKH must equal b because segment KG spans two squares in the same way CJ does. Just like angle ABC, angle CKG must measure 45°, since triangle CKG is a right triangle.



Activity Synthesis

The goal of this discussion is for students to see different equations that can be used to represent and solve for the same unknown angle measures.

Select students to share their answers to each problem. Consider asking some of the following questions:

- "Did anyone use a different equation for this same problem? If so, did you get the same answer?"
- "Were any of the questions harder than others? Why?"
- "Were there any questions for which you used a strategy that was new to you?"

Access for English Language Learners

MLR8 Discussion Supports. Pair gestures with verbal directions to clarify the meaning of any unfamiliar terms, such as "complementary," "supplementary," and "vertical."
Advances: Listening, Representing

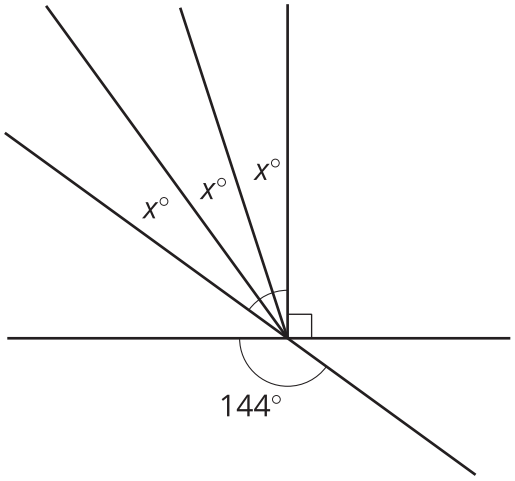
Lesson Synthesis

Here are some questions for discussion:

- How can equations help us solve for an unknown angle measure? (They allow us to represent relationships among angles. Then we can solve the equations to find the unknown angle measures.)
- Is there only one way to solve for an unknown angle measure? (No, there are usually a few different equations that can be used, based on the relationships present in the figure.)

Lesson 18 Summary

To find an unknown angle measure, sometimes it is helpful to write and solve an equation that represents the situation. For example, suppose we want to know the value of x in this diagram.



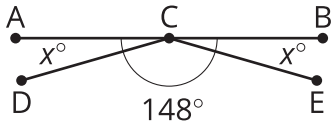
Using what we know about vertical angles, we can write the equation $3x + 90 = 144$ to represent this situation. Then we can solve the equation.

$$\begin{aligned} 3x + 90 &= 144 \\ 3x + 90 - 90 &= 144 - 90 \\ 3x &= 54 \\ 3x \cdot \frac{1}{3} &= 54 \cdot \frac{1}{3} \\ x &= 18 \end{aligned}$$

Lesson 18 Practice Problems

1 Student Task Statement

Segments AB, DC, and EC intersect at point C. Angle DCE measures 148°. Find the value of x.

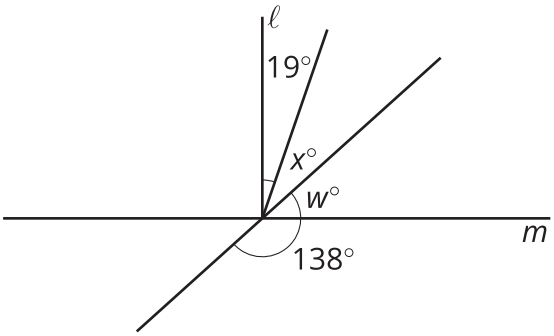


Solution

16

2 Student Task Statement

Line ℓ is perpendicular to line m. Find the value of x and w.



Solution

$x = 29$ and $w = 42$

3 Student Task Statement

If you knew that two angles were complementary and were given the measure of one of those angles, would you be able to find the measure of the other angle? Explain your reasoning.

Solution

Yes, because one angle would be known, and if two angles are complementary, then the measures of the two angles have a sum of 90°.

4 from Unit 3, Lesson 15

Student Task Statement

For each inequality, decide whether the solution is represented by $x < 4.5$ or $x > 4.5$.

a. $-24 > -6(x - 0.5)$

b. $-8x + 6 > -30$

c. $-2(x + 3.2) < -15.4$

Solution

a. $x > 4.5$

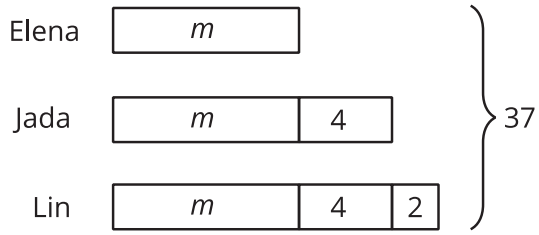
b. $x < 4.5$

c. $x > 4.5$

5 from Unit 3, Lesson 11

Student Task Statement

Jada, Elena, and Lin walked a total of 37 miles last week. Jada walked 4 more miles than Elena, and Lin walked 2 more miles than Jada. The diagram represents this situation:



Find the number of miles that they each walked. Explain or show your reasoning.

Solution

Elena: 9 miles, Jada: 13 miles, Lin: 15 miles

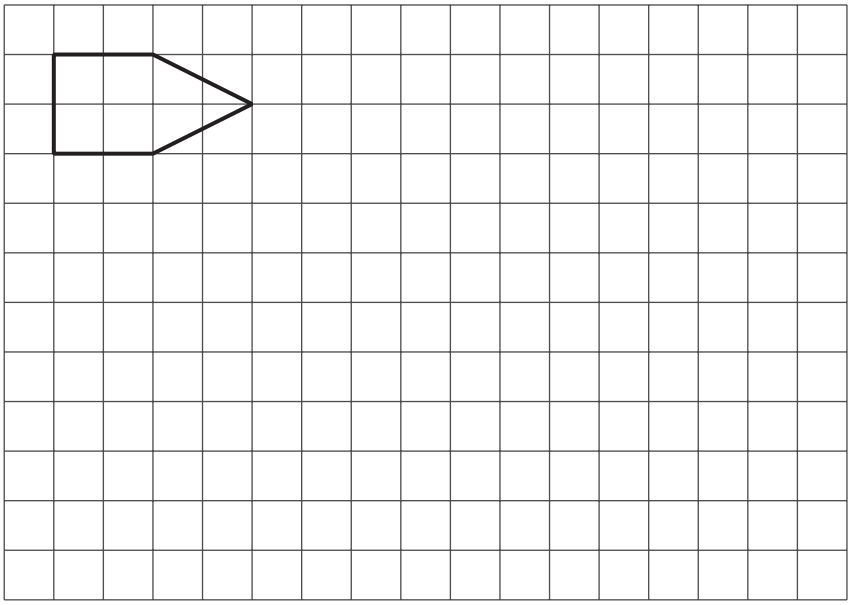
Possible strategies:

- $3m + 10 = 37$, $m = 9$
- Start with the total of 37 miles, subtract 10, and divide by 3

6 from Unit 2, Lesson 2

Student Task Statement

Here is a polygon on a grid.



a. Draw a scaled copy of the polygon using a scale factor 3. Label the copy A.

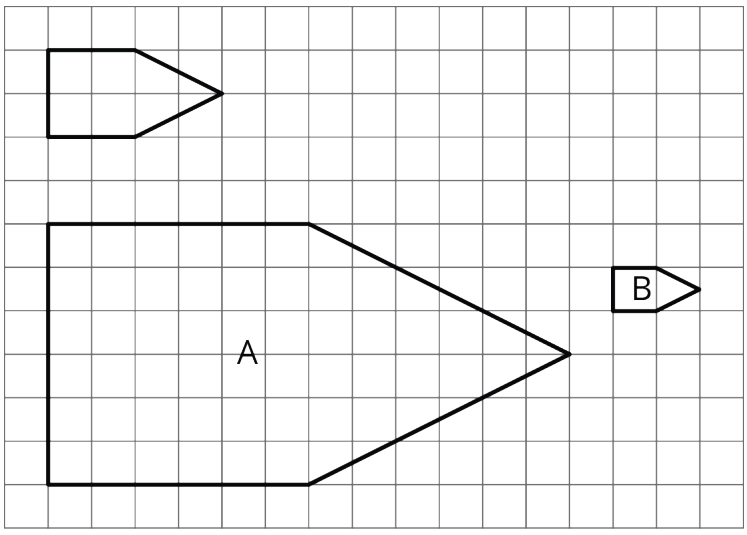
b. Draw a scaled copy of the polygon with a scale factor $\frac{1}{2}$. Label it B.

c. Is Polygon A a scaled copy of Polygon B? If so, what is the scale factor that takes B to A?

Solution

a. See grid.

b. See grid.



c. Yes, A is a scaled copy of B with a scale factor of 6.