Lesson 5: Reasoning About Square Roots

Let's approximate square roots.

5.1: True or False: Squared

Decide if each statement is true or false.

$$\left(\sqrt{5}\right)^2 = 5 \qquad\qquad\qquad \left(\sqrt{10}\right)^2 = 100$$

$$\left(\sqrt{9}\right)^2 = 3 \qquad\qquad\qquad \left(\sqrt{16}\right) = 2^2$$

$$7 = \left(\sqrt{7}\right)^2$$

5.2: Square Root Values

What two whole numbers does each square root lie between? Be prepared to explain your reasoning.

1. $\sqrt{7}$
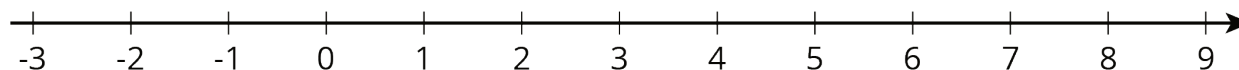
2. $\sqrt{23}$

3. $\sqrt{50}$

4. $\sqrt{98}$

Are you ready for more?

Can we do any better than "between 3 and 4" for $\sqrt{12}$? Explain a way to figure out if the value is closer to 3.1 or closer to 3.9.

5.3: Solutions on a Number Line

The numbers x, y, and z are positive, and $x^2 = 3$, $y^2 = 16$, and $z^2 = 30$.
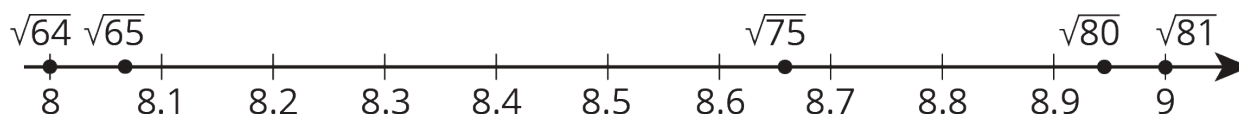
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 ← |——+——|——+——|——+——|——+——|——+——|——+——|——+——|——+——|——+——| →
   -3    -2    -1     0     1     2     3     4     5     6     7     8     9
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1. Plot x, y, and z on the number line. Be prepared to share your reasoning with the class.

2. Plot $-\sqrt{2}$ on the number line.

Lesson 5 Summary

In general, we can approximate the values of square roots by observing the whole numbers around it, and remembering the relationship between square roots and squares. Here are some examples:

- $\sqrt{65}$ is a little more than 8, because $\sqrt{65}$ is a little more than $\sqrt{64}$ and $\sqrt{64} = 8$.

- $\sqrt{80}$ is a little less than 9, because $\sqrt{80}$ is a little less than $\sqrt{81}$ and $\sqrt{81} = 9$.

- $\sqrt{75}$ is between 8 and 9 (it's 8 point something), because 75 is between 64 and 81.

- $\sqrt{75}$ is approximately 8.67, because $8.67^2 = 75.1689$.

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√64 √65                                    √75                    √80  √81
  ●——●——+——+——+——+——+——+——●——+——+——+——●——+——● →
  8   8.1  8.2  8.3  8.4  8.5  8.6  8.7  8.8  8.9   9
```

If we want to find a square root between two whole numbers, we can work in the other direction. For example, since $22^2 = 484$ and $23^2 = 529$, then we know that $\sqrt{500}$ (to pick one possibility) is between 22 and 23.

Many calculators have a square root command, which makes it simple to find an approximate value of a square root.