

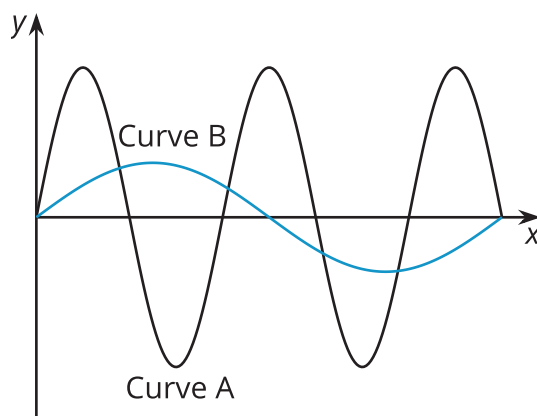


# Features of Trigonometric Graphs (Part 1)

Let's compare graphs and equations of trigonometric functions.

## 16.1 Musical Notes

Here are pictures of sound waves for two different musical notes:



## 16.2

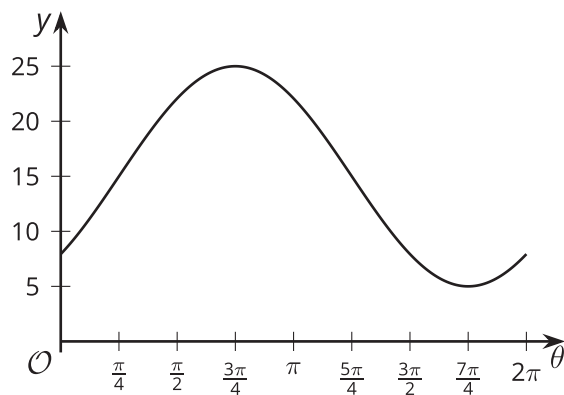
## Card Sort: Equations and Graphs

Your teacher will give you a set of cards. Take turns with your partner to match each graph with the equation and description that it represents. More than 1 equation or description can match the same graph.

1. For each match that you find, explain to your partner how you know it's a match.
2. For each match that your partner finds, listen carefully to your partner's explanation. If you disagree, discuss your thinking and work to reach an agreement.

**Are you ready for more?**

1. Use the sine function to find an equation for this graph.
2. Use the cosine function to find another equation for the same graph.

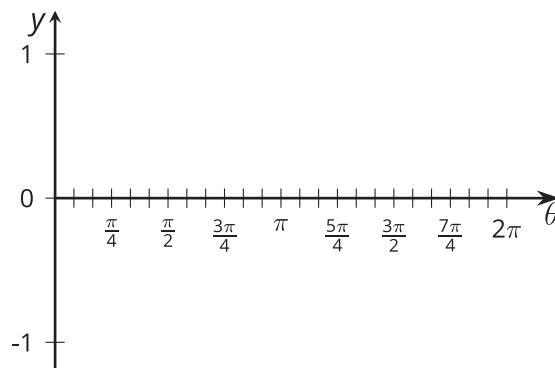


## 16.3 Double the Sine

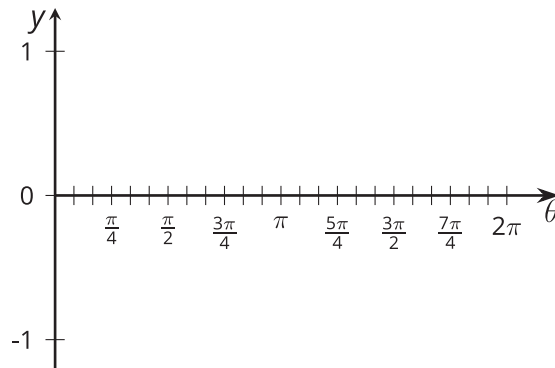
1. Complete the table of values for the expression  $\sin(2\theta)$

| $\theta$        | 0 | $\frac{\pi}{12}$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\frac{3\pi}{4}$ | $\pi$ | $\frac{5\pi}{4}$ | $\frac{3\pi}{2}$ | $\frac{7\pi}{4}$ | $2\pi$ |
|-----------------|---|------------------|-----------------|-----------------|-----------------|------------------|-------|------------------|------------------|------------------|--------|
| $\sin(2\theta)$ |   |                  |                 |                 |                 |                  |       |                  |                  |                  |        |

2. Plot the values and sketch a graph of the equation  $y = \sin(2\theta)$ . How does the graph of  $y = \sin(2\theta)$  compare to the graph of  $y = \sin(\theta)$ ?

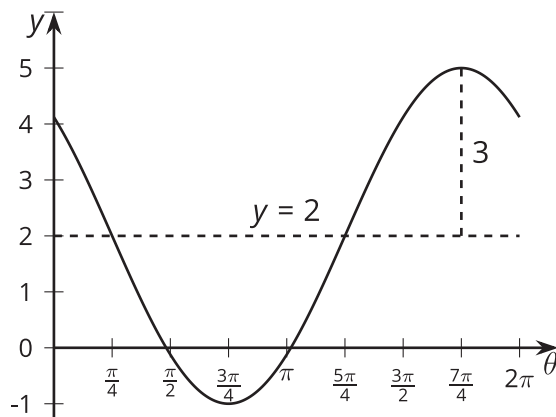


3. Predict what the graph of  $y = \cos(4\theta)$  will look like and make a sketch. Explain your reasoning.

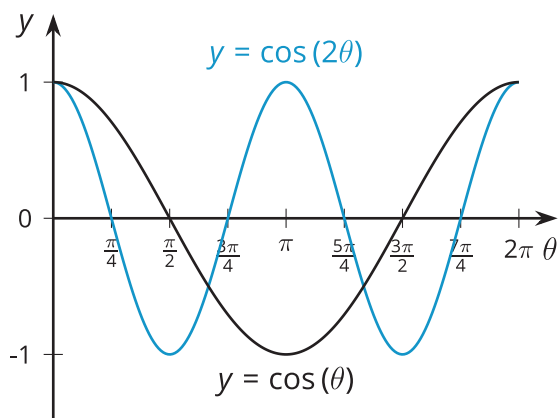


## Lesson 16 Summary

We can find the amplitude and midline of a trigonometric function using the graph or from an equation. For example, let's look at the function given by the equation  $y = 3 \cos\left(\theta + \frac{\pi}{4}\right) + 2$ . We can see that the midline of this function is 2 because of the vertical translation up by 2. This means the horizontal line  $y = 2$  goes through the middle of the graph. The amplitude of the function is 3. This means that the maximum value it takes is 5, 3 more than the midline value, and the minimum value it takes is -1, 3 less than the midline value. The horizontal translation is  $\frac{\pi}{4}$  to the left, so instead of having, for example, a minimum at  $\pi$ , the minimum is at  $\frac{3\pi}{4}$ . Here is what the graph looks like:



Another type of transformation is one that affects the period, and that is when a horizontal scale factor is used. For example, let's look at the equation  $y = \cos(2\theta)$ , where the variable,  $\theta$ , is multiplied by a number. Here, 2 is the scale factor affecting  $\theta$ . When  $\theta = 0$ , we have  $2\theta = 0$  so the graph of this cosine equation starts at  $(0, 1)$ , just like the graph of  $y = \cos(\theta)$ . When  $x = \pi$ , we have  $2\theta = 2\pi$ , so the graph of  $y = \cos(2\theta)$  goes through two full periods in the same horizontal span that it takes  $y = \cos(\theta)$  to complete one full period, as shown in their graphs.



Notice that the graph of  $y = \cos(2\theta)$  has the same general shape as the graph of  $y = \cos(\theta)$  (same midline and amplitude) but the waves are compressed together. And what if we wanted to give the graph of cosine a stretched appearance? Then we could use a horizontal scale factor between 0 and 1. For example, the graph of  $y = \cos\left(\frac{\theta}{6}\right)$  has a period of  $12\pi$ .