Unit 8, Lesson 8

Rewriting Quadratic Expressions in Factored Form (Part 3)

Let's look closely at some special kinds of factors.

8.1 Math Talk: Products of Large-ish Numbers

Evaluate mentally.

- $9 \cdot 11$
- $19 \cdot 21$
- $99 \cdot 101$
- $119 \cdot 121$

8.2 Can Products Be Written as Differences?

1. Clare claims that $(10 + 3)(10 - 3)$ is equivalent to $10^2 - 3^2$ and $(20 + 1)(20 - 1)$ is equivalent to $20^2 - 1^2$. Do you agree? Show your reasoning.

2. a. Use your observations from the first question to evaluate $(100 + 5)(100 - 5)$. Show your reasoning.

   b. Check your answer by computing $105 \cdot 95$.

3. Is $(x + 4)(x - 4)$ equivalent to $x^2 - 4^2$? Support your answer:

   With a diagram:

	x	4
x		
-4		

   Without a diagram:

4. Is $(x + 4)^2$ equivalent to $x^2 + 4^2$? Support your answer, either with or without a diagram.

Are you ready for more?

1. Explain how your work in the previous questions can help you mentally evaluate $22 \cdot 18$ and $45 \cdot 35$.

2. Here is a shortcut that can be used to mentally square any two-digit number. Let's take 83^2, for example.

   - 83 is $80 + 3$.
   - Compute 80^2 and 3^2, which give 6,400 and 9. Add these values to get 6,409.
   - Compute $80 \cdot 3$, which is 240. Double it to get 480.
   - Add 6,409 and 480 to get 6,889.

   Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

8.3 What If There Is No Linear Term?

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram. (Heads up: One of them is impossible.)

factored form	standard form
$(x - 10)(x + 10)$	
$(2x + 1)(2x - 1)$	
$(4 - x)(4 + x)$	
	$x^2 - 81$
	$49 - y^2$
	$9z^2 - 16$
	$25t^2 - 81$
$(c + \frac{2}{5})(c - \frac{2}{5})$	
	$\frac{49}{16} - d^2$
$(x + \sqrt{5})(x - \sqrt{5})$	
	$x^2 - 6$
	$x^2 + 100$

Lesson 8 Summary

Sometimes expressions in standard form don't have a linear term. Can they still be written in factored form?

Let's take $x^2 - 9$ as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0: $x^2 + 0x - 9$.

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is $(x + 3)(x - 3)$.

To check that this expression is indeed equivalent to $x^2 - 9$, we can expand the factored expression by applying the distributive property: $(x + 3)(x - 3) = x^2 - 3x + 3x + (\text{-}9)$. Adding $\text{-}3x$ and $3x$ gives 0, so the expanded expression is $x^2 - 9$.

In general, a quadratic expression that is a difference of two squares and has the form $a^2 - b^2$ can be rewritten as $(a + b)(a - b)$.

Here is a more complicated example: $49 - 16y^2$. This expression can be written as $7^2 - (4y)^2$, so an equivalent expression in factored form is $(7 + 4y)(7 - 4y)$.

What about $x^2 + 9$? Can it be written in factored form?

Let's think about this expression as $x^2 + 0x + 9$. Can we find two numbers that multiply to make 9 and add up to 0? Here are factors of 9 and their sums:

- 9 and 1, sum: 10
- -9 and -1, sum: -10
- 3 and 3, sum: 6
- -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write $x^2 + 9$ in factored form using the kinds of numbers that we know about.