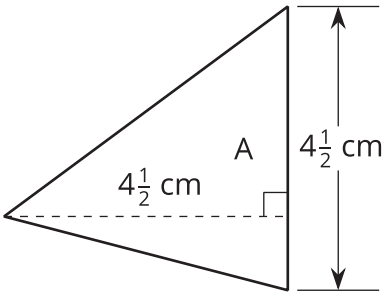
Unit 4, Lesson 14

Fractional Lengths in Triangles and Prisms

Let's explore area and volume when fractions are involved.

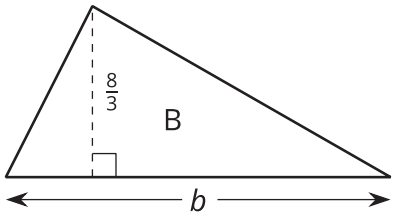
14.1 Area of Triangle

Find the area of Triangle A in square centimeters.
Show your reasoning.

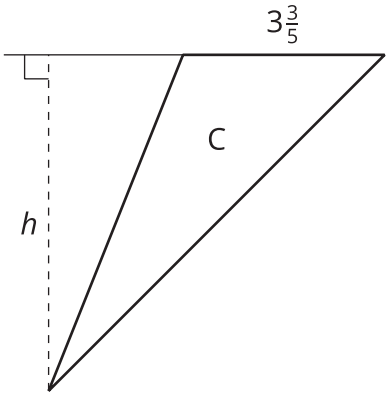


14.2 Bases and Heights of Triangles

1. The area of Triangle B is 8 square units. Find the length of b.
   Show your reasoning.



2. The area of Triangle C is $\frac{54}{5}$ square units.
   What is the length of h? Show your reasoning.

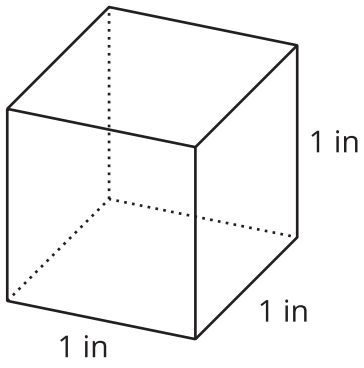


14.3 Volumes of Cubes and Prisms

Your teacher will give you cubes that have edge lengths of $\frac{1}{2}$ inch.

1. Here is a drawing of a cube with edge lengths of 1 inch.

   a. How many cubes with edge lengths of $\frac{1}{2}$ inch are needed to fill this cube?



   b. What is the volume, in cubic inches, of a cube with edge lengths of $\frac{1}{2}$ inch? Explain or show your reasoning.

2. Four cubes are piled in a single stack to make a prism. Each cube has an edge length of $\frac{1}{2}$ inch. Sketch the prism, and find its volume in cubic inches.

3. Use cubes with an edge length of $\frac{1}{2}$ inch to build prisms with the lengths, widths, and heights shown in the table.

   For each prism, record in the table how many $\frac{1}{2}$-inch cubes can be packed into the prism and the volume of the prism.

prism length (in)	prism width (in)	prism height (in)	number of $\frac{1}{2}$-inch cubes in prism	volume of prism (in^3)
$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$		
1	1	$\frac{1}{2}$		
2	1	$\frac{1}{2}$		
2	2	1		
4	2	$\frac{3}{2}$		
5	4	2		
5	4	$2\frac{1}{2}$		

Are you ready for more?

A unit fraction has a 1 in the numerator.

- These are unit fractions: $\frac{1}{3}, \frac{1}{100}, \frac{1}{1}$.
- These are *not* unit fractions: $\frac{2}{9}, \frac{8}{1}, 2\frac{1}{5}$.

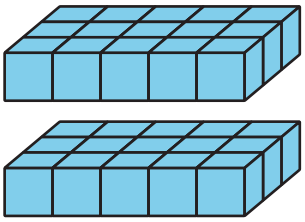
1. Find three unit fractions whose sum is $\frac{1}{2}$. An example is: $\frac{1}{8} + \frac{1}{8} + \frac{1}{4} = \frac{1}{2}$. How many examples like this can you find?

2. Find a box whose surface area in square units equals its volume in cubic units. How many like this can you find?

Lesson 14 Summary

If a rectangular prism has edge lengths of 2 units, 3 units, and 5 units, we can think of it as 2 layers of unit cubes, with each layer having $(3 \cdot 5)$ unit cubes in it. So the volume, in cubic units, is:

$$2 \cdot 3 \cdot 5$$



To find the volume of a rectangular prism with fractional edge lengths, we can think of it as being built of cubes that have a unit fraction for their edge length. For instance, if we build a prism that is $\frac{1}{2}$-inch tall, $\frac{3}{2}$-inch wide, and 4 inches long using cubes with a $\frac{1}{2}$-inch edge length, we would have:

- A height of 1 cube, because $1 \cdot \frac{1}{2} = \frac{1}{2}$.
- A width of 3 cubes, because $3 \cdot \frac{1}{2} = \frac{3}{2}$.
- A length of 8 cubes, because $8 \cdot \frac{1}{2} = 4$.

The volume of the prism would be $1 \cdot 3 \cdot 8$, which is 24 cubic units.

How do we find its volume in cubic inches? We know that each cube with a $\frac{1}{2}$-inch edge length has a volume of $\frac{1}{8}$ cubic inch, because $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$. Since the prism is built using 24 of these cubes, its volume, in cubic inches, would then be $24 \cdot \frac{1}{8}$, which is 3 cubic inches.

The volume of the prism, in cubic inches, can also be found by multiplying the fractional edge lengths in inches: $\frac{1}{2} \cdot \frac{3}{2} \cdot 4 = 3$