



# A Radical Identity

Let's use an identity with square roots.

## 13.1 Quadratic Zeros

1. Let  $f(x) = (x - 3)(x - 4)$ .
  - a. For what values of  $x$  is the function  $f$  equal to 0? Recall that these values are called *zeros*.
  - b. Rewrite the equation in standard form,  $f(x) = ax^2 + bx + c$ .
  - c. For this function, how are  $b$  and  $c$  related to the zeros?
  - d. Is this the only quadratic function with zeros at 3 and 4?
2. Let  $g(x) = (x - s)(x - t)$ .
  - a. What are the zeros of  $g$ ?
  - b. Rewrite the equation in standard form.

## 13.2 An Identity from Zeros

Let  $s$  and  $t$  be the two solutions from the quadratic formula.

$$s = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

1. Multiply  $s$  and  $t$  to find a simple expression in terms of  $a$ ,  $b$ , and  $c$ .
2. Add  $s$  and  $t$  to find a simple expression in terms of  $a$ ,  $b$ , and  $c$ .
3. Show that  $a(x - s)(x - t) = ax^2 + bx + c$ .
4. Use the connection between the variables  $s$ ,  $t$ ,  $b$ , and  $c$  to write the quadratic function  $f(x)$  in standard form that has zeros at  $x = 1 + \sqrt{5}$  and  $x = 1 - \sqrt{5}$  and a quadratic coefficient of 1 ( $a = 1$ ). Explain or show your reasoning.



### Are you ready for more?

What about complex solutions? Let  $s = m + ni$  and  $t = m - ni$  for real numbers  $m$  and  $n$  and imaginary number  $i$ .

Write a function in standard form that represents all quadratic functions with zeros at  $s$  and  $t$  in terms of  $m$  and  $n$ .

## 13.3 Removing Square Roots

1. Finish this identity:  $d \cdot 1 = \underline{\hspace{2cm}}$ . Are there any restrictions on what  $d$  can be for the identity to be true?
2. Finish this identity:  $\frac{c}{c} = \underline{\hspace{2cm}}$ . Are there any restrictions on what  $c$  can be for the identity to be true?

Pause here for a discussion.

3. Show that  $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

4. Show that  $\frac{2}{\sqrt{5} + \sqrt{3}} = \sqrt{5} - \sqrt{3}$

## Lesson 13 Summary

Mathematical identities can be useful in a variety of situations.

In an earlier course, we learned how to derive the quadratic formula using the method of completing the square on a quadratic equation. We can check the formula by reconstructing the standard form of a quadratic equation using the zeros guaranteed by the quadratic formula. We can start with writing the factored form using the zeros as

$$a \left( x - \frac{-b + \sqrt{b^2 - 4ac}}{2a} \right) \left( x - \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right) = 0.$$

After distributing the left side of the equation, it nicely becomes  $ax^2 + bx + c = 0$ .

Identities can also be used to rewrite expressions to be in a form that is clearer to understand. For example, the value of the fraction  $\frac{5}{\sqrt{7} + \sqrt{2}}$  is not very easy to estimate without a calculator. Let's

rewrite the expression using two identities: the difference of squares identity,  $(a + b)(a - b) = a^2 - b^2$ , and the identity  $\frac{c}{c} = 1$  when  $c \neq 0$ .

$$\begin{aligned} \frac{5}{\sqrt{7} + \sqrt{2}} &= \frac{5}{\sqrt{7} + \sqrt{2}} \cdot \frac{\sqrt{7} - \sqrt{2}}{\sqrt{7} - \sqrt{2}} \\ &= \frac{5(\sqrt{7} - \sqrt{2})}{(\sqrt{7})^2 - (\sqrt{2})^2} \\ &= \frac{5(\sqrt{7} - \sqrt{2})}{7 - 2} \\ &= \sqrt{7} - \sqrt{2} \end{aligned}$$

In this form, we can estimate that  $\sqrt{7}$  is approximately 2.5 and  $\sqrt{2}$  is a little more than 1, so the expression has a value of approximately 1.25. Checking on a calculator, we can see that the original fraction and the final expression are equal and have a value of approximately 1.232, which is close to our estimation.