Solving Radical Equations

Let’s practice solving radical equations.

10.1 Math Talk: Radical Equations

Solve these equations mentally:

- $\sqrt[3]{x} = 1$
- $\sqrt{7} = \sqrt{x - 1}$
- $\sqrt{100} = 2x$
- $\sqrt{x + 1} = -5$

10.2 Rectangles and Boxes

1. A rectangle has its length twice as long as its width.
   a. Write an equation for the area, A, of the rectangle in terms of the width, w.
   b. If you know that the value for A is positive, how many solutions are there for w? Explain your reasoning.
   c. If the area of the rectangle is 100 square inches, what is the exact width of the rectangle?
2. A rectangular prism has a square base and a height equal to 4 times the length of one of the base sides.
   a. Write an equation for the length of the side of the base, s, in terms of the volume of the prism, V.
   b. If you know the volume of the rectangular prism, how many solutions are there for s? Explain your reasoning.
   c. If the volume of the rectangular prism is 8 cubic meters, what is the exact length of the side of the base?
3. Kiran has a small scale model of an airplane that is $\frac{1}{200}$ of the actual size of an airplane.
   a. Kiran measures the area of the wings and finds that it is about 95 square centimeters. What is the area of the actual airplane wings?
   b. He uses water displacement to find that the volume of the model is about 16 cubic centimeters. What is the volume of the actual airplane?

10.3 Write Your Own Equation

1. Write an equation that includes a radical symbol with:
   a. one solution

   b. no solutions

   c. two solutions

2. Switch with a partner and solve their equations.

Are you ready for more?

Find all solutions to the equation $\sqrt{x} = \sqrt[3]{x}$. Explain how you know those are all of the solutions.

Lesson 10 Summary

Areas of figures are often related to two lengths multiplied together, and volumes of objects are often related to three lengths multiplied together. If the lengths are related, this can result in an equation with powers of 2 or 3. Here are some examples:

- The area of a circle with radius r is $A = \pi \cdot r^2$.
- The area of an equilateral triangle with side length s is $A = \frac{\sqrt{3}}{4} \cdot s^2$.
- The volume of a sphere with radius r is $V = \frac{4\pi}{3} \cdot r^3$.
- The volume of a regular tetrahedron (a pyramid made from 4 equilateral triangles) with side length s is $V = \frac{1}{6\sqrt{2}} \cdot s^3$.

We can solve each of these equations for r or s to give us an equation that would allow us to find the radius or side length from the area or volume. These equations would include either a square root or cube root.

Whenever we have an equation with a radical symbol that contains a variable, we can solve it by isolating the radical and then raising each side of the equation to a power in order to get a new equation without radicals. Here is an example:

$$\begin{aligned} -4 &= \sqrt[3]{5p+1} \\ (-4)^3 &= \left(\sqrt[3]{5p+1}\right)^3 \\ -64 &= 5p+1 \\ -65 &= 5p \\ -13 &= p \end{aligned}$$

Sometimes this results in an equation with solutions that do not make the original equation true. If we use this strategy, it is good to check the solutions to the new equation after raising each side to a power, to be sure they make the original equation true. In this example, we did find a solution to the original equation because $\sqrt[3]{5(-13)+1} = -4$.

Another way to solve these equations is to reason about what the answer is, instead of raising each side to a power. For example, if we are solving $\sqrt{1-x} + 5 = 11$, we can rearrange it to get $\sqrt{1-x} = 6$ and then think, "If the positive square root of $1-x$ is 6, then $1-x$ must be 36, because the positive square root of 36 is 6. So x must be -35, because $1-(-35) = 36$." If we check this result, we see that -35 is a solution to the original equation because $\sqrt{1-(-35)} + 5 = 11$.