



# Solving Exponential Equations

Let's solve equations using logarithms.

## 14.1 A Valid Solution?

Here is a solution to the equation  $5 \cdot e^{3a} = 90$ .

$$\begin{aligned}5 \cdot e^{3a} &= 90 \\e^{3a} &= 18 \\3a &= \log_e 18 \\a &= \frac{\log_e 18}{3}\end{aligned}$$

Explain what happened in each step.

## 14.2 Natural Logarithm

1. Complete the table with equivalent equations. The first row is completed for you.

|    | exponential form       | logarithmic form         |
|----|------------------------|--------------------------|
|    | $e^6 \approx 403.43$   | $\ln(403.43) \approx 6$  |
| a. | $e^0 = 1$              |                          |
| b. | $e^1 = e$              |                          |
| c. | $e^{-1} = \frac{1}{e}$ |                          |
| d. |                        | $\ln \frac{1}{e^2} = -2$ |
| e. | $e^x = 10$             |                          |

2. Solve each equation by expressing the solution using  $\ln$  notation. Then, find the approximate value of the solution using the “ln” button on a calculator.

a.  $e^m = 20$

b.  $e^n = 30$

c.  $e^p = 7.5$

## 14.3

## Solving Exponential Equations

Without using a calculator, solve each equation. It is expected that some solutions will be expressed using log notation. Be prepared to explain your reasoning.

1.  $10^x = 10,000$

2.  $5 \cdot 10^x = 500$

3.  $10^{(x+3)} = 10,000$

4.  $10^{2x} = 10,000$

5.  $10^x = 315$

6.  $2 \cdot 10^x = 800$

7.  $10^{(1.2x)} = 4,000$

8.  $7 \cdot 10^{(0.5x)} = 70$

9.  $2 \cdot e^x = 16$

10.  $10 \cdot e^{3x} = 250$



 **Are you ready for more?**

Assume that  $a$  and  $b$  are positive values. Use your understanding of what logarithms mean to find these values. Explain or show your reasoning.

1.  $\log_a(a^b)$

2.  $a^{\log_a(b)}$

## Lesson 14 Summary

So far we have solved exponential equations by

- Finding whole number powers of the base (for example, the solution to  $10^x = 100$  is  $x = 2$ , and the solution to  $10^y = 1,000$  is  $y = 3$ ).
- Estimation (for example, the solution of  $10^x = 300$  is between 2 and 3 because 300 is between 100 and 1,000).
- Using a logarithm and approximating its value on a calculator (for example, the solution of  $10^x = 300$  is  $\log 300 \approx 2.48$ ).

Sometimes solving exponential equations takes additional reasoning. Here are a couple of examples.

$$5 \cdot 10^x = 45$$

$$10^x = 9$$

$$x = \log 9$$

$$10^{(0.2t)} = 1,000$$

$$10^{(0.2t)} = 10^3$$

$$0.2t = 3$$

$$t = \frac{3}{0.2}$$

$$t = 15$$

In the first example, the power of 10 is multiplied by 5, so to find the value of  $x$  that makes this equation true, each side is divided by 5. From there, the equation is rewritten as a logarithm, giving an exact value for  $x$ .

In the second example, the expressions on each side of the equation are rewritten as powers of 10:  $10^{(0.2t)} = 10^3$ . This means that the exponent  $0.2t$  on one side and the 3 on the other side must be equal, and leads to an expression to solve where we don't need to use a logarithm.

How do we solve an exponential equation with base  $e$ , such as  $e^x = 5$ ? We can express the solution using the **natural logarithm**, the logarithm for base  $e$ . Natural logarithm is written as  $\ln$ , or sometimes as  $\log_e$ . Just like the equation  $10^2 = 100$  can be rewritten, in logarithmic form, as  $\log_{10} 100 = 2$  or  $\log 100 = 2$ , the equation  $e^0 = 1$  can be rewritten as  $\ln 1 = 0$ . Similarly,  $e^{-2} = \frac{1}{e^2}$  can be rewritten as  $\ln \frac{1}{e^2} = -2$ .

All this means that we can solve  $e^x = 5$  by rewriting the equation as  $x = \ln 5$ . This says that  $x$  is the exponent to which base  $e$  is raised to equal 5.

To estimate the size of  $\ln 5$ , remember that  $e$  is about 2.7. Because 5 is greater than  $e^1$ , this means that  $\ln 5$  is greater than 1.  $e^2$  is about  $(2.7)^2$ , or 7.3. Because 5 is less than  $e^2$ , this means that  $\ln 5$  is less than 2. This suggests that  $\ln 5$  is between 1 and 2. Using a calculator we can check that  $\ln 5 \approx 1.61$ .