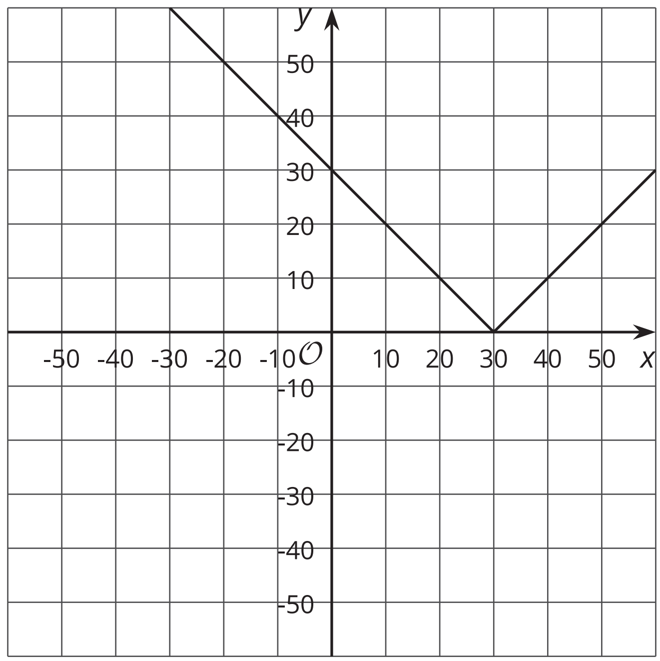
Solving Equations with Absolute Values

Let’s solve absolute value equations.

15.1 Where Does 30 Go?

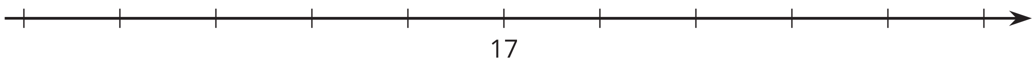


1. Which of these functions is represented by this graph? Explain your reasoning.
   - $f(x) = |x + 30|$
   - $g(x) = |x - 30|$
   - $h(x) = |x| + 30$
   - $j(x) = |x| - 30$
2. One of these functions takes a guess as an input, x, and outputs the distance between the guess and 30 as the output. Which function is it? Explain your reasoning.
3. Write a function that takes a guess as an input and outputs the distance between the guess and -8.

15.2 Using Distance to Solve Absolute Value Equations

1. $|x - 17| = 3$

   What values of x make the equation true? Explain or show your reasoning.



2. $|12 - t| = 3$. What values of t make the equation true? Explain or show your reasoning.

3. Write an equation that uses absolute value and represents all the numbers that are exactly 4 away from 12.

Are you ready for more?

1. What number is exactly halfway between the numbers 34 and 86 on a number line? Explain or show your reasoning.

2. How can you use that value to write an absolute value equation where the solutions are 34 and 86? Explain or show your reasoning.

15.3 Using Cases to Solve Absolute Value Equation

Andre solves the equation $2|x-47|+5=9$ by first getting the absolute value by itself on one side of the equation.

$2|x-47|=4$

$|x-47|=2$

Then he uses the piecewise definition to split the equation into two cases.

- If $x \geq 47$, then $|x-47|=(x-47)$, so the equation becomes $x-47=2$, which can be solved to $x=49$. This fits with the initial condition because $49 \geq 47$.
- If $x<47$, then $|x-47|=-(x-47)$, so the equation becomes $-(x-47)=2$, which can be solved to $x=45$. This fits with the initial condition because $45<47$.

Solve 1 of these equations using Andre's method of using cases. Solve a different equation by reasoning about distances on a number line. Solve the other 2 equations using either method.

1. $|x-16|=20$
2. $|x+9|+2=4$
3. $|7-x|=9$
4. $|x-4|=\frac{1}{2}$

Are you ready for more?

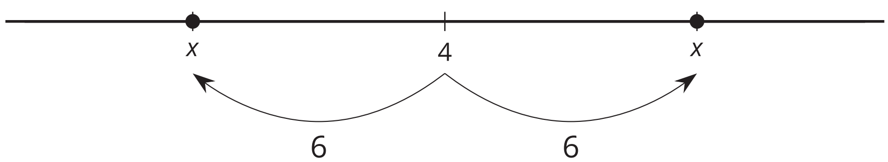
Use Andre’s method to solve these equations. How can you explain the answers in terms of distance to a number?

1. $|2x-3|=5$
2. $2|x-1|=10$

Lesson 15 Summary

The statement $|3 - 5| = 2$ is true and can be interpreted as "The distance between 3 and 5 is 2." This is why it is also true to write $|5 - 3| = 2$.

The idea can also be used to solve equations in which a distance is known to a point. For example, $|x - 4| = 6$ tells us that the distance between x and 4 is 6. On a number line, it might look like this.



What values of x make this statement true? Both $x = -2$ and $x = 10$ make the equation true, so they are solutions to the absolute value equation.

Another way to solve an equation involving an absolute value is to look at the piecewise meaning.

$$|x - a| = \begin{cases} (x - a), & x \geq a \\ -(x - a), & x < a \end{cases}$$

We can take the equation $|x - 4| = 6$ and split it into two cases.

- If $x \geq 4$, then the equation becomes $(x - 4) = 6$. We can solve this using what we learned about linear equations. Adding 4 to each side, we can rewrite the equation as $x = 10$. We can check that this is fine for this case because the case begins with "If $x \geq 4...$" and ends with a value for x that is greater than or equal to 4.
- If $x < 4$, then the equation becomes $-(x - 4) = 6$. We can distribute the -1 on the left to get $-x + 4 = 6$. From there, we can subtract 4 from each side and multiply each side of the result by -1 to get $x = -2$. We can also check that this is fine for this case because the case begins with "If $x < 4...$" and ends with a value for x that is less than 4.

Putting the results of the cases together, we again get that the solutions are -2 and 10.

Whether you prefer reasoning about distances on a number line or using the piecewise definition to write equations in different cases, the solutions should be the same.

