Using Probability to Determine Whether Events Are Independent

Let’s take a closer look at dependent and independent events.

10.1 Which Three Go Together: Events

A coin is flipped and a standard number cube is rolled. Which three sets go together? Why do they go together?

Set 1

Event A1: the coin landing heads up

Event B1: rolling a 3 or 5

Set 2

Event A2: rolling a 3 or 5

Event B2: rolling an odd number

Set 3

Event A3: rolling a prime number

Event B3: rolling an even number

Set 4

Event A4: the coin landing heads up

Event B4: the coin landing tails up

10.2 Overtime Wins

Does this hockey team perform differently in games that go into overtime (or shootout) compared to games that don't? The table shows data about the team over 5 years.

Let A represent the event "the hockey team wins a game" and B represent "the game goes to overtime or shootout."

year	games played	total wins	overtime or shootout games played	wins in overtime or shootout games
2018	82	46	19	6
2017	82	46	18	7
2016	82	51	23	16
2015	82	54	18	10
2014	82	34	17	5
total	410	231	95	44

1. Use the data to estimate the probabilities. Explain or show your reasoning.
   a. $P(\text{A})$
   b. $P(\text{B})$
   c. $P(\text{A and B})$
   d. $P(\text{A} \mid \text{B})$
2. We have seen two ways to check for independence using probability. Use your estimates to check whether each might be true.
   a. $P(\text{A} \mid \text{B}) = P(\text{A})$
   b. $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$
3. Based on these results, do you think the events are independent?

10.3 Genetic Testing

A group of scientists think that a variation in a certain gene contributes to the likelihood that a person gets a particular disease. A study gathers at-risk people at random and tests them for the disease as well as for the genetic variation.

	has the disease	does not have the disease
has the genetic variation	80	12
does not have the genetic variation	1,055	1,160

A person from the study is selected at random. Let A represent the event "has the disease" and B represent "has the genetic variation."

1. Use the table to find the probabilities. Show your reasoning.
   a. $P(\text{A})$
   b. $P(\text{B})$
   c. $P(\text{A and B})$
   d. $P(\text{A} \mid \text{B})$
2. Based on these probabilities, are the events independent? Explain your reasoning.
3. From your analysis, do you think there is evidence to support the scientists' theory that this genetic variation contributes to the likelihood that a person gets the disease?
4. A company that tests for this genetic variation has determined that someone has the variation and wants to inform the person that they may be at risk of developing this disease when they get older. Based on this study, what percentage chance of getting the disease should the company report as an estimate to the person? Explain your reasoning.

Are you ready for more?

Find or collect your own data that can be represented using a two-way table. Some ideas include finding data about a sports team (shooting percentage greater than 50% or less than 50% vs. winning or losing) or clinical trials for a medication (placebo or medication vs. symptom or no symptom).

1. Create a two-way table to represent the sample space.

2. Use probabilities to determine whether events summarized in the table are dependent or independent events.

3. When only a little data is collected in an experiment or a survey, it can be difficult to show evidence that events are independent. Why do you think this occurs? Explain your reasoning.

Lesson 10 Summary

Although it may not always be easy to determine whether events are dependent or independent based on their descriptions alone, there are several ways to check for independence using probabilities.

One way to recognize independence is by understanding the experiment well enough to see if it fits the definition:

- Events A and B are independent if the probability of Event A occurring does not change whether Event B occurs or not.

An example of independence that can be found this way might be the events "a coin landing heads up" and "rolling a 4 on a number cube" when flipping a coin and rolling a standard number cube. Whether the coin lands heads up or not does not change the probability of rolling a 4 on a number cube.

A second way to recognize independence is to use conditional probability:

- Events A and B are independent if $P(\text{A} \mid \text{B}) = P(\text{A})$

An example of independence that can be found this way might be the events "gets a hit on the second time at bat in a game" and "struck out in the first at bat in a game" for a baseball player in games for a season. By looking at what happens when the player has his second at bat, we can estimate that $P(\text{hit on second at bat}) = 0.331$ and by looking only at the second at bat after a strikeout, we can estimate that $P(\text{hit on second at bat} \mid \text{strike out on first at bat}) = 0.324$. Although the probabilities are not exactly the same, we might decide that being within 0.7% is close enough to equality, depending on the number of at bats in the data set, to say that these events are independent.

Another way to recognize independence is to look at the probability of both events happening:

- Events A and B are independent if $P(\text{A and B}) = P(\text{A}) \cdot P(\text{B})$

An example of independence that can be found this way might be the events "making the first free throw shot" and "making the second free throw shot" for a basketball player shooting two free throws after a foul. By looking at the outcomes of the two shots for the player throughout the year, we can estimate that $P(\text{make the first shot}) = 0.72$, $P(\text{make the second shot}) = 0.72$ and $P(\text{make the first shot and make the second shot}) = 0.52$. The events are independent since $0.72 \cdot 0.72 \approx 0.52$.