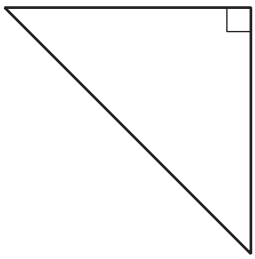
A Proof of the Pythagorean Theorem

Let's prove the Pythagorean Theorem.

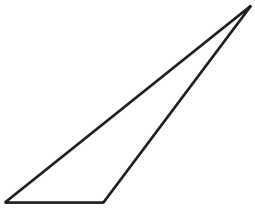
7.1 Which One Is the Hypotenuse?

Label all the hypotenuses with c.

A

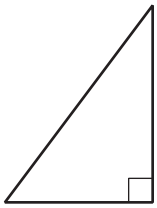


B



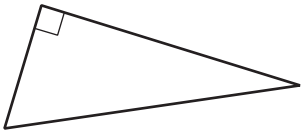
C

D



E

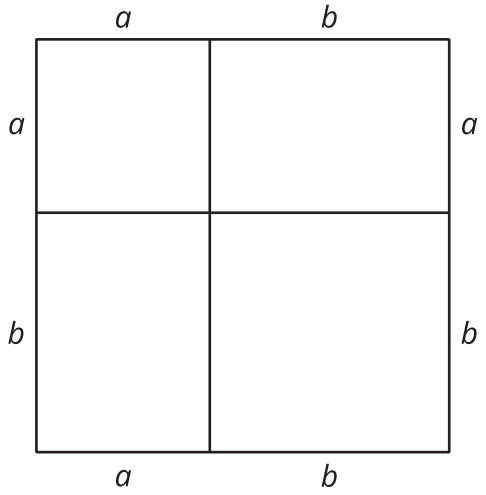
F



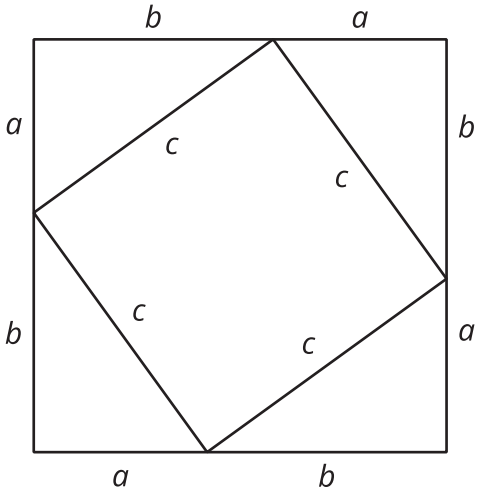
7.2 Adding Up Areas

Both figures shown here are squares with a side length of $a + b$. Notice that the first figure is divided into two squares and two rectangles. The second figure is divided into a square and four right triangles with **legs** of lengths a and b. Let's call the **hypotenuse** of these triangles c.

F



G



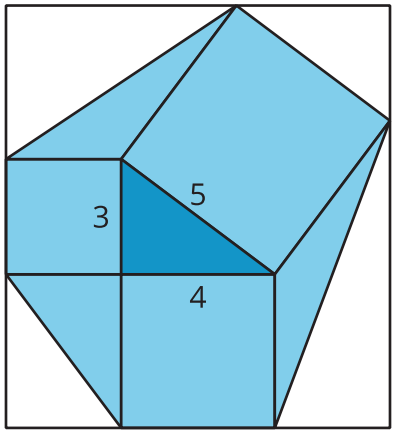
1. What is the total area of each figure?

2. Find the area of each of the 9 smaller regions shown in the figures and label them.

3. Add up the area of the 4 regions in Figure F and set this expression equal to the sum of the areas of the 5 regions in Figure G. If you rewrite this equation using as few terms as possible, what do you have?

Are you ready for more?

Take a 3-4-5 right triangle, add on the squares of the side lengths, and form a hexagon by connecting vertices of the squares as in the image. What is the area of this hexagon?



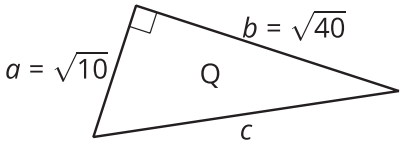
7.3 A Transformational Proof

Your teacher will give your group a sheet with 4 figures. Cut out the 5 shapes in Figure 1.

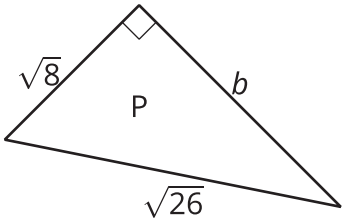
1. Arrange the 5 cut out shapes to fit inside Figure 2.
2. Now arrange the shapes to fit inside Figure 3.
3. Check to see that Figure 3 is congruent to the large square in Figure 4.
4. Check to see that the 5 cut out shapes fit inside the two smaller squares in Figure 4.
5. If the right triangle in Figure 4 has legs a and b and hypotenuse c, what have you just demonstrated to be true?

7.4 Find the Unknown Side Lengths

1. Find c.



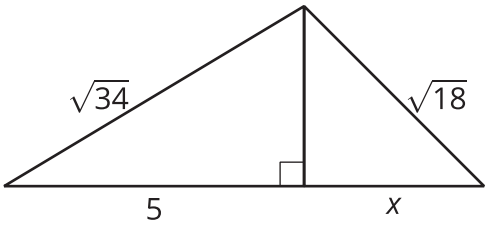
2. Find b.



3. A right triangle has sides of length 2.4 cm and 6.5 cm. What is the length of the hypotenuse?

4. A right triangle has a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$. What is the length of the other side?

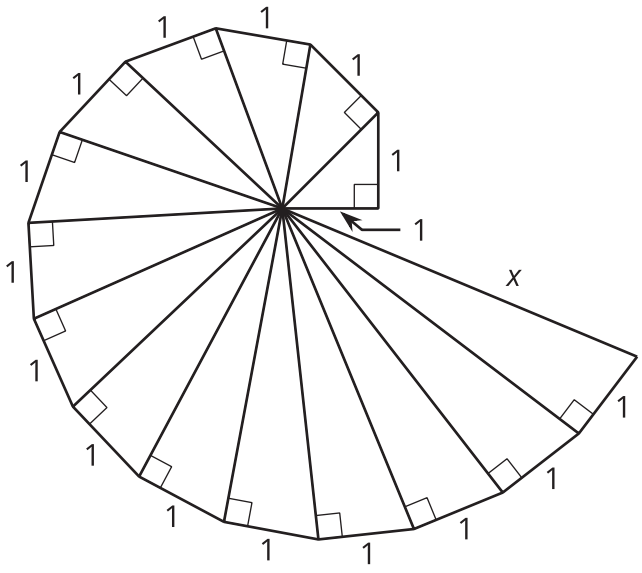
5. Find the value of x in the figure.



Are you ready for more?

The spiral in the figure is made by starting with a right triangle with both legs measuring 1 unit each. Then a second right triangle is built with one leg measuring 1 unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle's hypotenuse, again with the other leg measuring 1 unit, and so on.

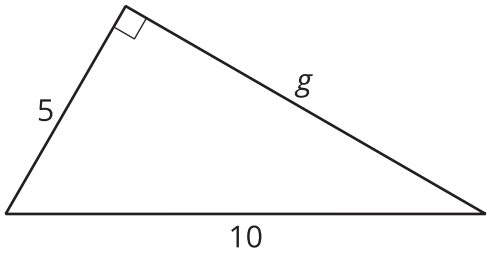
Find the length, x, of the hypotenuse of the last triangle constructed in the figure.



Lesson 7 Summary

The Pythagorean Theorem can be used to find an unknown side length in a right triangle as long as the length of the other two sides is known.

For example, here is a right triangle, where one leg has a length of 5 units, the hypotenuse has a length of 10 units, and the length of the other leg is represented by g.



Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse, the side opposite the right angle. For this triangle, the hypotenuse is 10.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
5^2 + g^2 &= 10^2 \\
g^2 &= 10^2 - 5^2 \\
g^2 &= 100 - 25 \\
g^2 &= 75 \\
g &= \sqrt{75}
\end{aligned}
$$

Use estimation strategies to know that the length of the other leg is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.