A Special Point

Let’s see what we can learn about a triangle by watching how salt piles up on it.

6.1 Notice and Wonder: Salt Pile

What do you notice? What do you wonder?

6.2 Point and Angle

Here is an angle BAC with two different sets of markings.

$\overline{EF} \perp \overline{AB}$;

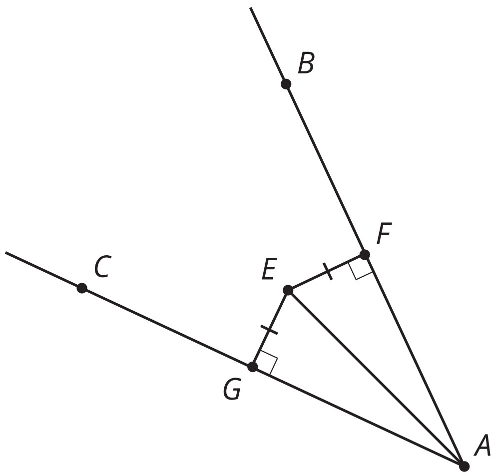
$\overline{EG} \perp \overline{AC}$;

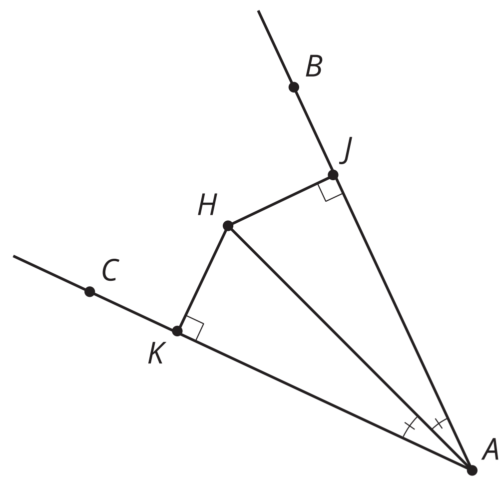
$\overline{EF} \cong \overline{EG}$

$\overline{HJ} \perp \overline{AB}$;

$\overline{HK} \perp \overline{AC}$;

$\angle BAH \cong \angle CAH$





1. Point E is the same distance away from each of the two rays that form angle BAC. Make a conjecture about angles EAB and EAC, and prove it.

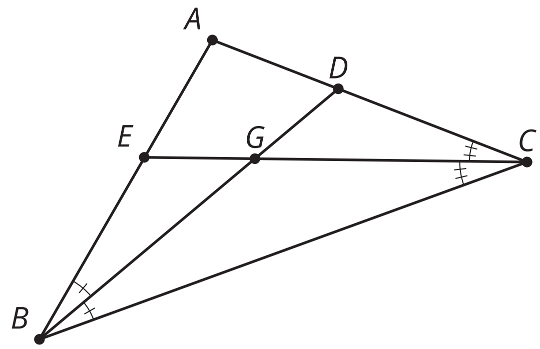
2. Point H is on the angle bisector of angle BAC. What can you prove about the distance from H to each ray?

6.3 What If There Are Three Sides?

Two angle bisectors have been constructed in triangle ABC. They intersect at point G.

$\angle DCG \cong \angle BCG$;

$\angle EBG \cong \angle CBG$



1. Sketch segments that show the distance from point G to each side of the triangle.
2. How do the distances from point G to sides AB and BC compare? Explain your reasoning.
3. How do the distances from point G to sides AC and BC compare? Explain your reasoning.
4. Will the third angle bisector pass through point G? Explain your reasoning.

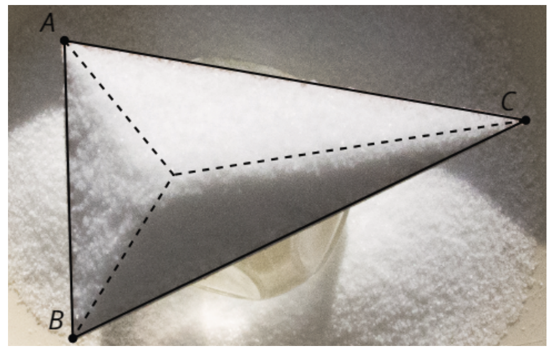
Are you ready for more?

What shape would the ridge form if you poured salt onto a piece of cardboard with a long straight edge and a small hole cut out of it? Explain your reasoning.

Lesson 6 Summary

Salt piles up in an interesting way when poured onto a triangle. Why does that happen?

As the salt piles up and reaches a maximum height, new grains of salt will fall off toward whichever side of the triangle is closest. We can show that points on an angle bisector are equidistant from the rays that form the angle. So salt grains that land on an angle bisector will balance and not fall toward either side. This is why we see ridges form in the salt.



As we might conjecture from the salt example, all three angle bisectors in a triangle meet at a single point, called the triangle's **incenter**. To see why this is true, consider any two angle bisectors in a triangle. The point where they meet is the same distance from the first and second sides, and is also the same distance from the second and third sides. Therefore, the point is the same distance from *all* sides, so the third angle bisector must also go through this point.

In the images, segments RT, AD, BD, and CD are angle bisectors. This means that, for angle QRS, point T is the same distance from ray RQ as it is from ray RS. In triangle ABC, point D is the same distance from all three sides of the triangle—it's the triangle's incenter.