



# Introducing the Number $i$

Let's meet  $i$ .

## 8.1 Math Talk: Squared

Find the value of each expression mentally.

$$\cdot (2\sqrt{3})^2$$

$$\cdot \left(\frac{1}{2}\sqrt{3}\right)^2$$

$$\cdot (2\sqrt{-1})^2$$

$$\cdot \left(\frac{1}{2}\sqrt{-1}\right)^2$$

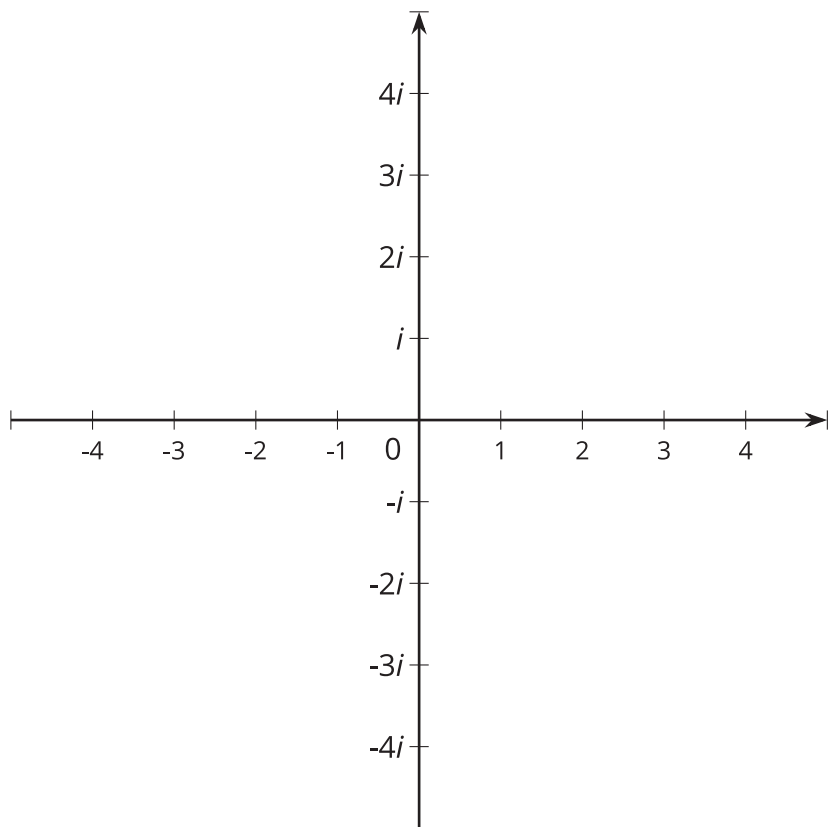
## 8.2 It Is $i$

Find the solutions to these equations, then plot the solutions to each equation on the imaginary or real number line.

1.  $a^2 = 16$

2.  $b^2 = -9$

3.  $c^2 = -5$



## 8.3 The $i$ 's Have It

Write these imaginary numbers using the number  $i$ .

1.  $\sqrt{-36}$

2.  $\sqrt{-10}$

3.  $-\sqrt{-100}$

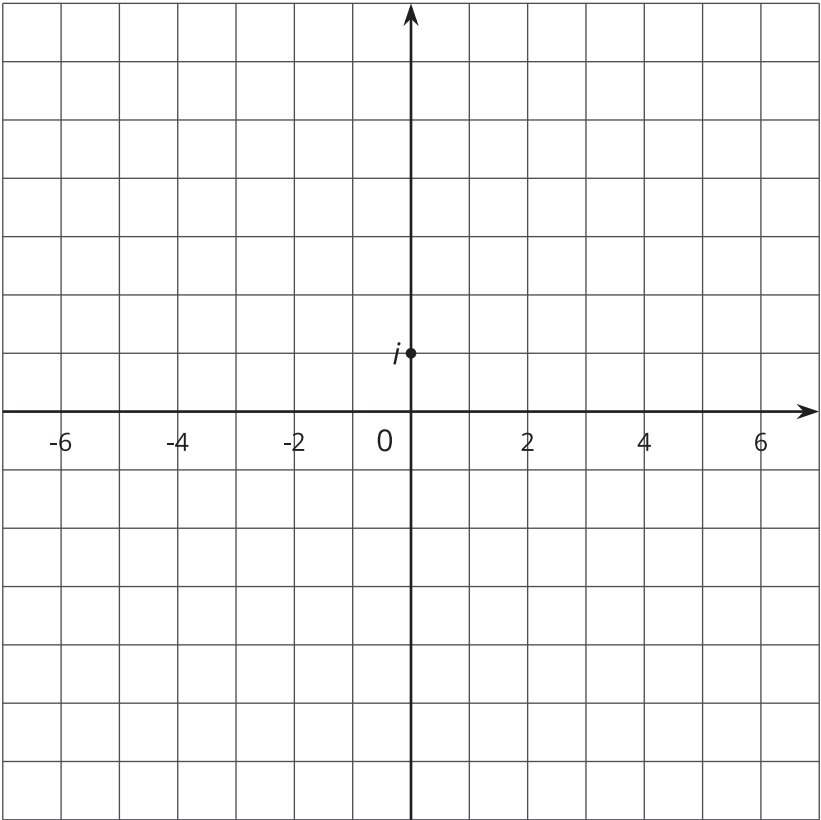
4.  $-\sqrt{-17}$



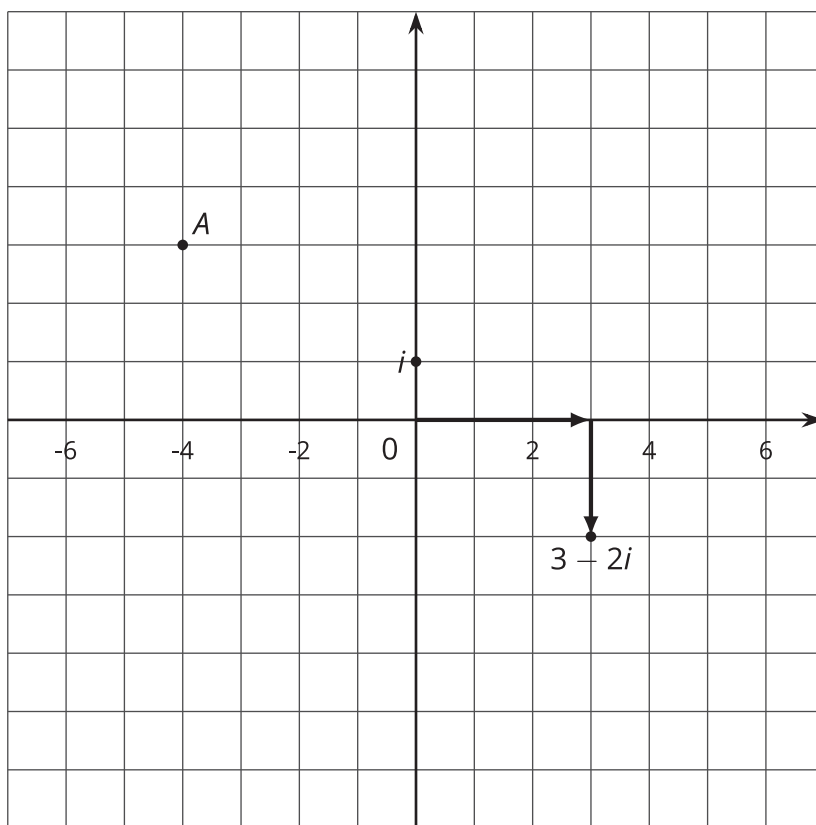
8.4

Complex Numbers

1. Label at least 8 different imaginary numbers on the imaginary number line.



2. When we add a real number and an imaginary number, we get a **complex number**. The diagram shows where  $3 - 2i$  is in the complex plane. What complex number is represented by point  $A$ ?



3. Plot these complex numbers in the complex number plane and label them.
- a.  $-2 - i$
  - b.  $-6 + 3i$
  - c.  $5 + 4i$
  - d.  $1 - 3i$

 **Are you ready for more?**

Diego says that all real numbers and all imaginary numbers are complex numbers but not all complex numbers are imaginary or real. Do you agree with Diego? Explain your reasoning.

## Lesson 8 Summary

A square root of a number  $a$  is a number whose square is  $a$ . In other words, it is a solution to the equation  $x^2 = a$ . Every positive real number has two *real* square roots. For example, the number 35 has both  $\sqrt{35}$  and  $-\sqrt{35}$  as square roots because those are the two numbers that square to make 35 (remember, the  $\sqrt{\quad}$  symbol together with a positive number is defined to indicate the *positive* square root). In other words,  $(\sqrt{35})^2 = 35$  and  $(-\sqrt{35})^2 = 35$ .

Similarly, every negative real number has two *imaginary* square roots. The two square roots of -1 are written  $i$  and  $-i$ . That means that  $i^2 = -1$  and  $(-i)^2 = -1$ .

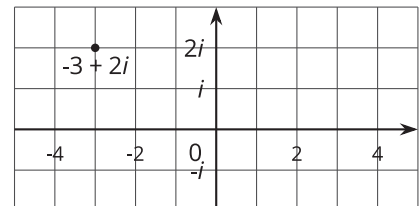
The two square roots of -17 are  $\sqrt{17}i$  and  $-\sqrt{17}i$ , because

$$\begin{aligned} (\sqrt{17}i)^2 &= 17i^2 && \text{and} && (-\sqrt{17}i)^2 &= 17i^2 \\ &= -17 && && &= -17 \end{aligned}$$

In general, if  $a$  is a positive real number, then the square roots of  $-a$  are  $\sqrt{a}i$  and  $-\sqrt{a}i$ .

Rarely, we might see something like  $\sqrt{-17}$ . By convention,  $\sqrt{-17}$  is defined to indicate the square root on the positive imaginary axis, so  $\sqrt{-17} = \sqrt{17}i$ .

When we add a real number and an imaginary number, we get a **complex number**. Together, the real number line and the imaginary number line form a coordinate system that can be used to represent any complex number as a point in the *complex plane*. For example, the point shown represents the complex number  $-3 + 2i$ .



In this context, people call the real number line the *real axis* and the imaginary number line the *imaginary axis*. This is different than the coordinate plane that you have seen before because those points were pairs of real numbers, like  $(-3, 2)$ , but in the complex plane, each point represents a single complex number. Note that since the real number line is part of the complex plane, real numbers are a special type of complex number. For example, the real number 5 can be described as the point  $5 + 0i$  in the complex plane.