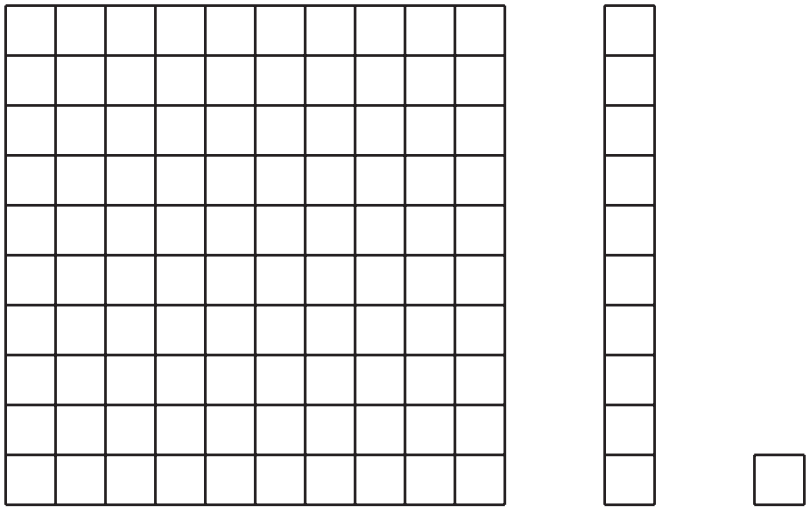
Multiplying Powers of 10

Let's explore patterns with exponents when we multiply powers of 10.

2.1 Picture a Power of 10

In the diagram, the medium rectangle is made up of 10 small squares. The large square is made up of 10 medium rectangles.



1. How could the large square be represented as a power of 10? Explain your reasoning.

2. If each small square represents 10^2, then what does the medium rectangle represent? The large square?

3. If each small square represents 10^5, then what does the medium rectangle represent? The large square?

2.2 Multiplying Powers of Ten

1. a. Complete the table to explore patterns in the exponents when multiplying powers of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^2 \cdot 10^3$	$(10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^5
$10^4 \cdot 10^3$		
$10^4 \cdot 10^4$		
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10 \cdot 10 \cdot 10)$	
$10^{18} \cdot 10^{23}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. a. Use the patterns you found in the table to rewrite $10^n \cdot 10^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

   b. Use your rule to write $10^4 \cdot 10^0$ with a single exponent. What does this tell you about the value of 10^0?

3. The state of Georgia has roughly 10^7 human residents. Each human has roughly 10^{13} bacteria cells in his or her digestive tract. How many bacteria cells are there in the digestive tracts of all the humans in Georgia?

Are you ready for more?

There are four ways to make 10^4 by multiplying powers of 10 with smaller, positive exponents.

$$10^1 \cdot 10^1 \cdot 10^1 \cdot 10^1$$

$$10^1 \cdot 10^1 \cdot 10^2$$

$$10^1 \cdot 10^3$$

$$10^2 \cdot 10^2$$

(This list is complete if you don't pay attention to the order you write them in. For example, we are only counting $10^1 \cdot 10^3$ and $10^3 \cdot 10^1$ once.)

1. How many ways are there to make 10^6 by multiplying smaller powers of 10 together?

2. How about 10^7? 10^8?

2.3 Raising Powers of 10 to Another Power

1. a. Complete the table to explore patterns in the exponents when raising a power of 10 to a power. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$(10^3)^2$	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	10^6
$(10^2)^5$	$(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)(10 \cdot 10)$	
	$(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)(10 \cdot 10 \cdot 10)$	
$(10^4)^2$		
$(10^8)^{11}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $(10^n)^m$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. If you took the amount of oil consumed in 2 months in 2013 worldwide, you could make a cube of oil that measures 10^3 meters on each side. How many cubic meters of oil is this? Do you think this would be enough to fill a pond, a lake, or an ocean?

Are you ready for more?

$2^{12} = 4{,}096$. How many other whole numbers can you raise to a power and get 4,096? Explain or show your reasoning.

Lesson 2 Summary

In this lesson, we developed a rule for multiplying powers of 10: Multiplying powers of 10 corresponds to adding the exponents together.

Rule	Example showing how it works
$10^n \cdot 10^m = 10^{n+m}$	$10^2 \cdot 10^3 = (10 \cdot 10) \cdot (10 \cdot 10 \cdot 10) = 10^5$ two factors that are ten · three factors that are ten = five factors that are ten

To see this, multiply 10^2 and 10^3. We know that 10^2 has two factors that are 10 and that 10^3 has three factors that are 10. That means that $10^2 \cdot 10^3$ has 5 factors that are 10.

This will work for other powers of 10, too. For example, $10^{14} \cdot 10^{47} = 10^{(14+47)} = 10^{61}$.

We also developed a rule for raising a power of 10 to another power: Taking a power of 10 and raising it to another power is the same as multiplying the exponents.

Rule	Example showing how it works
$(10^n)^m = 10^{n \cdot m}$	$(10^2)^3 = (10 \cdot 10) \cdot (10 \cdot 10) \cdot (10 \cdot 10) = 10^6$ three groups of two factors that are ten = six factors that are ten

To understand this, take 10^2 and raise it to the power of 3. We know that 10^2 has two factors that are 10. Raising 10^2 to the power of 3 means that there are three groups of two factors that are 10, for a total of 6 factors that are 10, or 10^6.

This works for any power of 10 raised to another power. For example, $(10^6)^{11} = 10^{(6 \cdot 11)} = 10^{66}$.