Unit 7, Lesson 4

Dividing Powers of 10

Let's explore patterns with exponents when we divide powers of 10.

4.1 A Surprising One

What is the value of the expression? Be prepared to explain your reasoning.

$$\frac{2^5 \cdot 3^4 \cdot 3^2}{2 \cdot 3^6 \cdot 2^4}$$

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

4.2 Dividing Powers of Ten

1. a. Complete the table to explore patterns in the exponents when dividing powers of 10. Use the "expanded" column to show why the given expression is equal to the single power of 10. You may skip a single box in the table, but if you do, be prepared to explain why you skipped it.

expression	expanded	single power of 10
$10^4 \div 10^2$	$\frac{10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10$	10^2
	$\frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10 \cdot 10 \cdot 10$	
$10^6 \div 10^3$		
$10^{43} \div 10^{17}$		

   b. If you chose to skip one entry in the table, which entry did you skip? Why?

2. Use the patterns you found in the table to rewrite $\frac{10^n}{10^m}$ as an equivalent expression with a single exponent, like $10^{\square}$.

3. It is predicted that by 2050, there will be 10^{10} people living on Earth. At that time, it is predicted there will be approximately 10^{12} trees. How many trees will there be for each person?

Are you ready for more?

expression	expanded	single power of 10
$10^4 \div 10^6$		

CC BY NC Illustrative Mathematics, based on IM 6–8 Math, CC BY Open Up Resources.

4.3 Zero Exponent

So far we have looked at powers of 10 with exponents greater than 0. Consider what would happen to our patterns if we included 0 as a possible exponent?

1. a. Write $10^{12} \cdot 10^{0}$ as a single power of 10. Explain or show your reasoning.

   b. What number could you multiply 10^{12} by to get this same answer?

2. a. Write $\frac{10^8}{10^0}$ as a single power of 10. Explain or show your reasoning.

   b. What number could you divide 10^8 by to get this same answer?

3. In order for the exponent rules we found to work even when the exponent is 0, what does the value of 10^0 have to be?

4.4 Making Millions

Write as many expressions as you can that have the same value as 10^8. Focus on using exponents, multiplication, and division.

Lesson 4 Summary

In this lesson, we developed a rule for dividing powers of 10: Dividing powers of 10 is the same as subtracting the exponent of the denominator from the exponent of the numerator. To see this, take 10^5 and divide it by 10^2.

Rule	Example showing how it works
$\frac{10^n}{10^m} = 10^{n-m}$	$\frac{10^5}{10^2} = \frac{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10}{10 \cdot 10} = \frac{10 \cdot 10}{10 \cdot 10} \cdot 10 \cdot 10 \cdot 10 = 1 \cdot 10^3 = 10^3$

five factors that are ten ÷ two factors that are ten = three factors that are ten

We know that 10^5 has 5 factors that are 10, and 2 of these factors can be divided by the 2 factors of 10 in 10^2 to make 1. That leaves $5 - 2 = 3$ factors of 10, or 10^3.

This will work for other powers of 10, too. For example $\frac{10^{56}}{10^{23}} = 10^{56-23} = 10^{33}$.

This rule also extends to 10^0. If we look at $\frac{10^6}{10^0}$, using the exponent rule gives 10^{6-0}, which is equal to 10^6. So dividing 10^6 by 10^0 doesn't change its value. That means if we want the rule to work when the exponent is 0, then 10^0 must equal 1.

Rule	Example showing how it works
$10^0 = 1$	$\frac{10^6}{10^0} = 10^{6-0} = 10^6$ this value must be equal to 1