



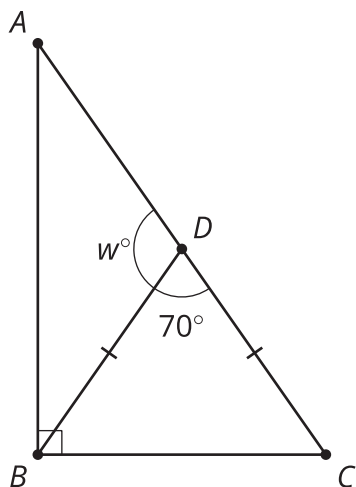
# Unknown Angles

Let's find the measures of unknown angles.

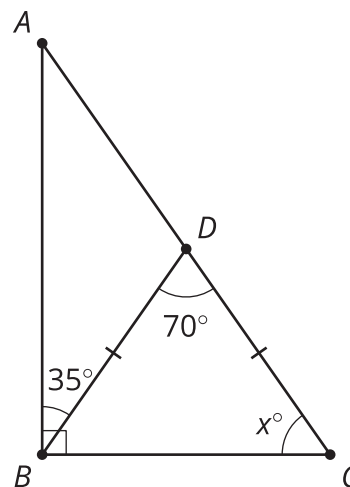
## 5.1 Math Talk: Unknown Angles

Find the value of each variable mentally.

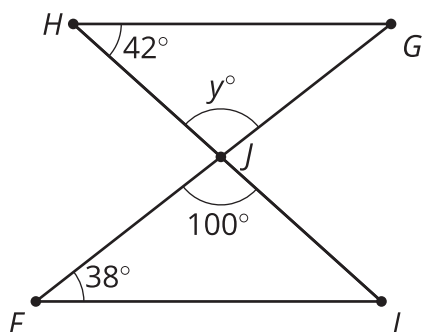
•  $\overline{BD} \cong \overline{CD}$



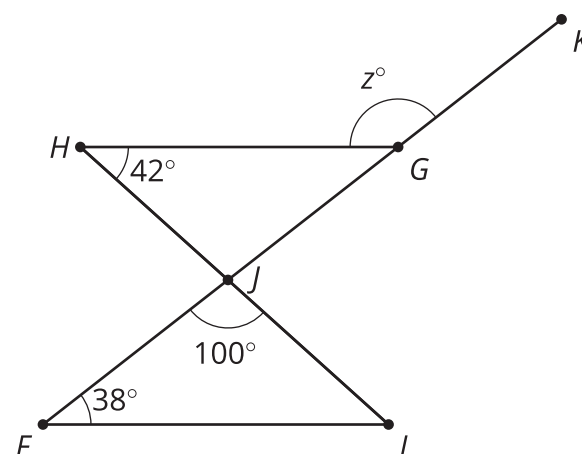
•  $\overline{BD} \cong \overline{CD}$



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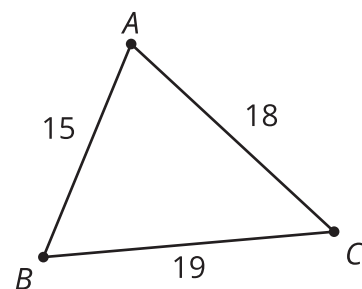


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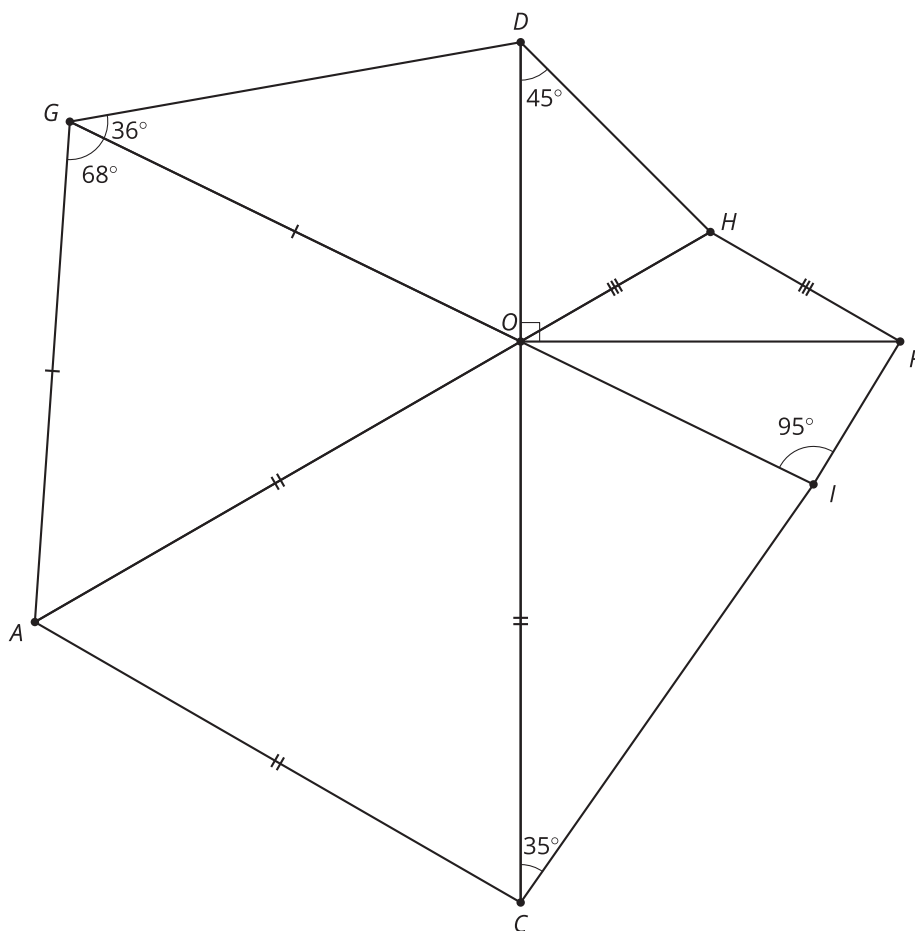
## 5.2 More Unknown Angles

- Here is triangle  $ABC$ . List the angles in order from smallest to largest.
- In triangle  $XYZ$  (not drawn), side  $XZ$  is the longest. Which angle is the largest? Explain how you know.



## 5.3 The Wacky Pinwheel

$$\overline{GO} \cong \overline{GA}, \overline{CO} \cong \overline{AO} \cong \overline{AC}, \overline{HO} \cong \overline{HF}, \overline{DO} \perp \overline{OF}$$



- Given that triangle  $GOA$  and triangle  $FOH$  are isosceles, triangle  $AOC$  is equilateral, and the measure of angle  $DOF$  is 90 degrees, label all unknown angle measures in the figure.

2. For each triangle listed in the table, name the longest side and shortest side.

triangle	longest side(s)	shortest side(s)
<i>AOC</i>		
<i>GOA</i>		
<i>DOG</i>		
<i>HOD</i>		
<i>FOH</i>		
<i>IOF</i>		
<i>COI</i>		

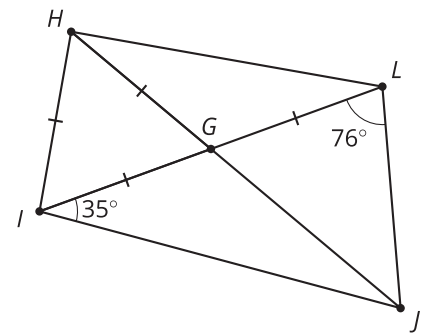
 **Are you ready for more?**

Is it possible to identify the longest side in the figure? What about the shortest side?

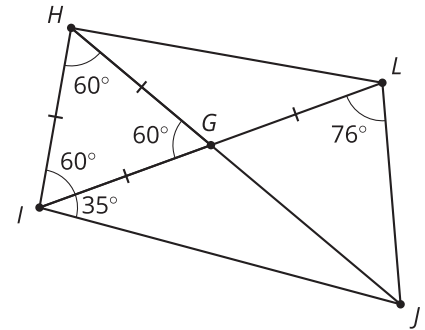
## Lesson 5 Summary

Suppose you need to find the unknown angles in a complicated figure and figure out which side is the longest. Here is a figure with several unknown angles.

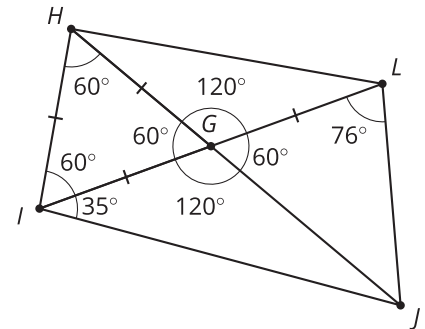
$$\overline{HI} \cong \overline{HG} \cong \overline{IG} \cong \overline{GL}$$



One way to start is to consider the equilateral triangle  $HGI$ . Since all sides are congruent, all angles in that triangle are congruent. Since the angles must add up to 180 degrees, each angle in triangle  $HGI$  is 60 degrees, because  $\frac{180}{3} = 60$ .

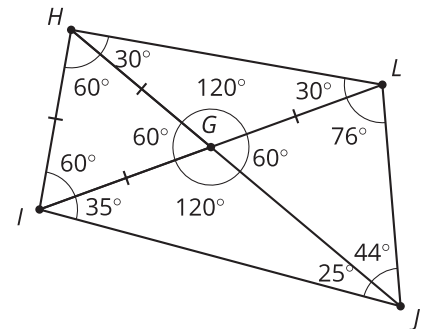


Next, consider the angles with a vertex at  $G$ . Using rules for supplementary and vertical angles, we can find these unknown angles. Angle  $HGL$  is supplementary to angle  $HGI$ , so the measure of angle  $HGL$  is 120 degrees because  $180 - 60 = 120$ . Since angle  $LGJ$  forms a vertical pair with angle  $HGI$ , these angles are congruent. Similarly, angle  $HGL$  forms a vertical pair with angle  $IGJ$ , so the measures of angles  $HGL$  and  $IGJ$  are equal.



Triangle  $HGL$  is isosceles, so we can use the Isosceles Triangle Theorem to find the measure of the two base angles. The measures of angles  $GHI$  and  $GLH$  are both 30 degrees because  $\frac{180-120}{2} = 30$ .

Finally, we can use the Triangle Angle Sum Theorem to find the two unknown angles. The measure of angle  $LJG$  is 45 degrees, because  $180 - (76 + 60) = 44$ , and the measure of angle  $GJI$  is 25 degrees, because  $180 - (120 + 35) = 25$ .



With this knowledge, we can say that  $HL$  is the longest side in triangle  $HGL$  and  $IJ$  is the longest side in triangle  $IGJ$ . This is not enough information to know which side is the longest overall or help us find the exact lengths of any of the sides.

