# Scope and Sequence for Grade 6

IM Grade 6 begins with an exploration of area and surface area—an invitation for students to engage with novel ideas that they can represent concretely and visually, and reason about in intuitive ways. Starting with geometry also creates opportunities to elicit close observation, sense- and connection-making, and the exchange of ideas—elements of a healthy learning community.

The next two units introduce ratios and rates, concepts that are also new. Students learn to represent, make sense of, and solve problems about equivalent ratios, rates, unit rates, and percentages. The mathematical reasoning here constitutes major work of the grade.

In the two units that follow, students expand and deepen their prior knowledge of numbers and operations. In one unit, students explore division involving fractions, and work toward dividing a fraction by fraction. In the other, they learn to multiply and divide multi-digit, base-ten numbers, including decimals, using the standard algorithm for each operation. Building fluency with algorithms takes time and continues beyond the two units.

Next, students further their understanding of equations and expressions, including those with variables. Students consider ways to represent, justify, and generate equivalent expressions. They also use expressions and equations to describe the relationship between quantities.

From there, students are introduced to rational numbers. Students learn about negative numbers, and represent negative numbers on the number line and on the coordinate plane. They analyze and write inequalities that compare rational numbers.

Toward the end of the course, students examine data sets and distributions. They learn about statistical questions, categorical data, and numerical data. They also explore ways to describe the center and the distribution of a data set.

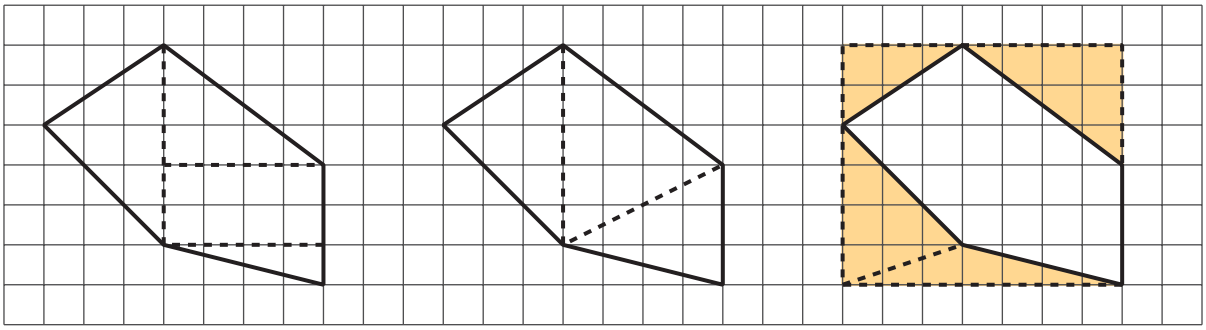
The final unit of the course is optional. The lessons provide students with additional opportunities to integrate and apply various ideas from the course to solve real-world and mathematical problems.

## Unit 1: Area and Surface Area

In this unit, students reason about areas of polygons and surface areas of polyhedra, building on geometric understandings developed in earlier grades.

In grade 3, students found the area of rectangles with whole-number side lengths. They also found the area of rectilinear figures by decomposing them into non-overlapping rectangles and adding those areas. Students used a formula for the area of rectangles in grade 4 and found the area of rectangles with fractional side lengths in grade 5.

In this unit, students extend their reasoning about area to include shapes that are not composed of rectangles. They use strategies such as decomposing and rearranging to find areas of parallelograms and generalize their process as a formula. Their work with parallelograms then becomes the basis for finding the area of triangles. Students see that other polygons can be decomposed into triangles and use this knowledge to find areas of polygons.



Next, students calculate the surface areas of polyhedra with triangular and rectangular faces. They study, assemble, and

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draw nets of prisms and pyramids and use nets to determine surface areas. Students also learn to use exponents $^2$ and $^3$ to express surface areas and volumes of cubes and their units.

In many lessons, students engage in geometric work without a context. This design choice is made in recognition of the significant intellectual work of reasoning about area. Later in the unit, students have opportunities to apply their learning in context.

Students will draw on the work here to further study exponents later in grade 6 and to find volumes of prisms and pyramids in grade 7. Their understanding of "two figures that match up exactly" will support their work on congruence and rigid motions in grade 8.

*A note about multiplication notation:*

Students in grade 6 will be writing algebraic expressions and equations involving the letter $x$. Because $x$ is easily confused with the "cross" notation for multiplication, $\times$, these materials use the "dot" notation for multiplication. Starting a few lessons into the unit, students will see, for instance, $2 \cdot 3$ instead of $2 \times 3$. The notation will be new to many students, so they will need explicit guidance in using it.

*A note about tools:*

Students are likely to need physical tools to support their reasoning. For instance, they may find that tracing paper is an excellent tool for verifying that figures "match up exactly." At all times in the unit, each student should have access to a geometry toolkit, which contains tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles. Access to the toolkit also enables students to practice selecting appropriate tools and using them strategically (MP5). In a digitally enhanced classroom, apps and simulations should be considered additions to their toolkits, not replacements for physical tools.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as comparing, explaining, and describing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Compare**

- Geometric patterns and shapes (Lesson 1).
- Strategies for finding areas of shapes (Lesson 3) and polygons (Lesson 11).
- The characteristics of prisms and pyramids (Lesson 13).
- The measurements and units of 1-, 2-, and 3-dimensional attributes (Lesson 16).
- Representations of area and volume (Lesson 17).

**Explain**

- How to find areas by composing (Lesson 3).
- Strategies used to find areas of parallelograms (Lesson 4) and triangles (Lesson 8).
- How to determine the area of a triangle using its base and height (Lesson 9).
- Strategies to find surface areas of polyhedra (Lesson 14).

**Describe**

- Observations about decomposition of parallelograms (Lesson 7).
- Information needed to find the surface area of rectangular prisms (Lesson 12).
- The features of polyhedra and their nets (Lesson 13).
- The features of polyhedra (Lesson 15).


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- Relationships among features of a tent and the amount of fabric needed for the tent (Lesson 19).

In addition, students are expected to justify claims about the base, height, or area of shapes; generalize about the features of parallelograms and polygons; interpret relevant information for finding the surface area of rectangular prisms; and represent the measurements and units of 2- and 3-dimensional figures. Over the course of the unit, teachers can support students' mathematical understandings by amplifying (not simplifying) language used for all of these purposes as students demonstrate and develop ideas.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.1.1 | area<br>**region**<br>plane<br>gap<br>overlap | |
| 6.1.2 | **area**<br>compose<br>decompose<br>rearrange<br>two-dimensional | |
| 6.1.3 | shaded<br>strategy | |
| 6.1.4 | parallelogram<br>opposite (sides or angles) | quadrilateral |
| 6.1.5 | **base (of a parallelogram or triangle)**<br>**height**<br>corresponding<br>expression<br>represent | |
| 6.1.6 | horizontal<br>vertical | |
| 6.1.7 | identical | parallelogram |
| 6.1.8 | diagram | **base (of a parallelogram or triangle)**<br>**height**<br>**compose**<br>**decompose**<br>rearrange |
| 6.1.9 | **opposite vertex** | |
| 6.1.10 | **vertex**<br>**edge** | |
| 6.1.11 | **polygon** | horizontal<br>vertical |
| 6.1.12 | **face**<br>**surface area** | **area**<br>**region** |


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.1.13 | **polyhedron**<br>**net**<br>**prism**<br>**pyramid**<br>**base (of a prism or pyramid)**<br>three-dimensional | **polygon**<br>**vertex**<br>**edge**<br>**face** |
| 6.1.15 | | **prism**<br>**pyramid** |
| 6.1.16 | **volume**<br>appropriate<br>quantity | two-dimensional<br>three-dimensional |
| 6.1.17 | **squared**<br>**cubed**<br>**exponent**<br>edge length | |
| 6.1.18 | value (of an expression) | **squared**<br>**cubed**<br>**net** |
| 6.1.19 | estimate<br>description | **surface area**<br>**volume** |

## Section A: Reasoning to Find Area

- Lesson 1: Tiling the Plane
- Lesson 2: Finding Area by Decomposing and Rearranging
- Lesson 3: Reasoning to Find Area

## Section B: Parallelograms

- Lesson 4: Parallelograms
- Lesson 5: Bases and Heights of Parallelograms
- Lesson 6: Area of Parallelograms

## Section C: Triangles and Other Polygons

- Lesson 7: From Parallelograms to Triangles
- Lesson 8: Area of Triangles
- Lesson 9: Formula for the Area of a Triangle
- Lesson 10: Bases and Heights of Triangles
- Lesson 11: Polygons


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### Section D: Surface Area

- Lesson 12: What Is Surface Area?
- Lesson 13: Polyhedra
- Lesson 14: Nets and Surface Area
- Lesson 15: More Nets, More Surface Area
- Lesson 16: Distinguishing Between Surface Area and Volume

### Section E: Squares and Cubes

- Lesson 17: Squares and Cubes
- Lesson 18: Surface Area of a Cube

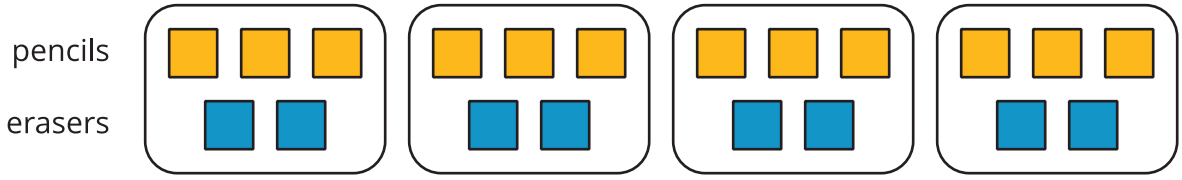
### Section F: Let's Put It to Work

- Lesson 19: All about Tents

## Unit 2: Introducing Ratios

This unit introduces students to ratios and equivalent ratios. It builds on previous experiences students had with relating two quantities, such as converting measurements starting in grade 3, multiplicative comparison in grade 4, and interpreting multiplication as scaling in grade 5. The work prepares students to reason about unit rates and percentages in the next unit, proportional relationships in grade 7, and linear relationships in grade 8.

First, students learn that a ratio is an association between two quantities, for instance, "There are 3 pencils for every 2 erasers." Students use sentences, drawings, or discrete diagrams to represent ratios that describe collections of objects and recipes.




Next, students encounter equivalent ratios in terms of multiple batches of a recipe. "Equivalent" is first used to describe a perceivable sameness of two ratios, such as two mixtures of drink mix and water that taste the same, or two mixtures of yellow and blue paint that make the same shade of green. Later, "equivalent" acquires a more precise meaning: All ratios that are equivalent to $a : b$ can be made by multiplying both $a$ and $b$ by the same non-zero number (non-negative, for now).

Students then learn to use double number line diagrams and tables to represent and reason about equivalent ratios. These representations are more abstract than are discrete diagrams and offer greater flexibility. Use of tables here is a stepping stone toward use of tables to represent functional relationships in future courses. Students explore equivalent ratios in contexts such as constant speed and uniform pricing.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*A note on using the terms "quantity," "ratio," "rate," and "proportion":*

In these materials, a "quantity" is a measurement that can be specified by a number and a unit, for instance, 4 oranges, 4 centimeters, “my height in feet,” or “my height” (with the understanding that a unit of measurement will need to be chosen).

The term "ratio" is used to mean an association between two or more quantities. In this unit, the fractions $\frac{a}{b}$ and $\frac{b}{a}$ are never called ratios, but the meanings of these fractions in contexts are very carefully developed. The word “per” is used with students in interpreting a unit rate in context, as in “$3 per ounce,” and the phrase “at the same rate” is used to signify a situation characterized by equivalent ratios. In the next unit, the fractions $\frac{a}{b}$ and $\frac{b}{a}$ will be identified as "unit rates" for the ratio $a : b$. Students will learn then that if two ratios $a : b$ and $c : d$ are equivalent, then the unit rates $\frac{a}{b}$ and $\frac{c}{d}$ are equal.

The terms "proportion" and "proportional" are not used in grade 6. A "proportional relationship" is a collection of equivalent ratios, which will be studied in grade 7. In high school—after their study of ratios, rates, and proportional relationships—students can discard the term “unit rate” and refer to $a$ to $b$, $a : b$, and $\frac{a}{b}$ all as “ratios.”

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Interpret**

- Statements and notations describing ratios (Lesson 2).
- Different representations of ratios (Lessons 2 and 6).
- Situations involving equivalent ratios (Lesson 8).
- Situations with different rates (Lesson 9).
- Tables of equivalent ratios (Lessons 11 and 12).
- Questions about situations involving ratios (Lesson 17).

**Explain**

- Reasoning about equivalence (Lesson 4).
- Reasoning about equivalent rates (Lesson 10).
- Reasoning with reference to tables (Lesson 14).
- Reasoning with reference to tape diagrams (Lesson 15).

**Compare**

- Situations with and without equivalent ratios (Lesson 3).
- Representations of ratios (Lessons 6 and 13).
- Situations with different rates (Lessons 9 and 12).


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- Situations with same rates and different rates (Lesson 10).
- Representations of ratio and rate situations (Lesson 16).

In addition, students are expected to describe and represent ratio associations, represent doubling and tripling of quantities in a ratio, represent equivalent ratios, justify whether ratios are or aren't equivalent and why information is needed to solve a ratio problem, generalize about equivalent ratios and about the usefulness of ratio representations, and critique representations of ratios.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.2.1 | **ratio**<br>___ to ___<br>___ for every ___ | |
| 6.2.2 | diagram | |
| 6.2.3 | recipe<br>batch<br>same taste<br>equivalent | **ratio**<br>___ to ___<br>___ for every ___ |
| 6.2.4 | mixture<br>same color<br>check (an answer) | batch |
| 6.2.5 | **equivalent ratios** | |
| 6.2.6 | **double number line diagram**<br>tick marks<br>representation | diagram |
| 6.2.7 | **per** | |
| 6.2.8 | **unit price**<br>how much for 1<br>at this rate | **double number line diagram** |
| 6.2.9 | **constant speed**<br>meters per second | |
| 6.2.10 | **same rate** | **equivalent ratios** |
| 6.2.11 | table<br>row<br>column | |
| 6.2.14 | calculation | **per**<br>table |
| 6.2.15 | **tape diagram**<br>parts<br>suppose | |
| 6.2.16 | | **tape diagram** |

## Section A: What Are Ratios?

- Lesson 1: Introducing Ratios and Ratio Language
- Lesson 2: Representing Ratios with Diagrams


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### Section B: Equivalent Ratios

- Lesson 3: Recipes
- Lesson 4: Color Mixtures
- Lesson 5: Defining Equivalent Ratios

### Section C: Representing Equivalent Ratios

- Lesson 6: Introducing Double Number Line Diagrams
- Lesson 7: Creating Double Number Line Diagrams
- Lesson 8: How Much for One?
- Lesson 9: Constant Speed
- Lesson 10: Comparing Situations by Examining Ratios

### Section D: Solving Ratio and Rate Problems

- Lesson 11: Representing Ratios with Tables
- Lesson 12: Navigating a Table of Equivalent Ratios
- Lesson 13: Tables and Double Number Line Diagrams
- Lesson 14: Solving Equivalent Ratio Problems

### Section E: Part-Part-Whole Ratios

- Lesson 15: Part-Part-Whole Ratios
- Lesson 16: Solving More Ratio Problems

### Section F: Let's Put It to Work

- Lesson 17: A Fermi Problem

## Unit 3: Unit Rates and Percentages

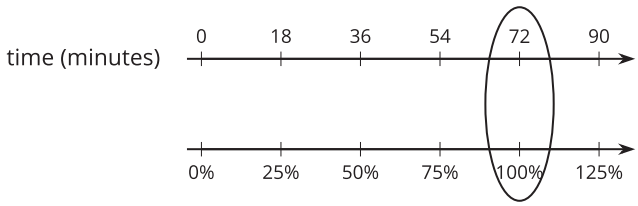
This unit develops students' understanding of unit rates and percentages. Students build on their experience with equivalent ratios and constant rates earlier in the course. They also build on knowledge of measurement and unit conversion in earlier grades. When learning about percentages, they draw on ideas about multiplicative comparison and equivalent fractions from grade 4 and multiplication of fractions from grade 5.

Students begin by recalling what they know about standard units of measurement—the attributes that they measure and their relative sizes. They use ratios and rates to reason about measurements and to convert between units of measurement.

Next, students learn about unit rates. They see that there are two unit rates—$\frac{a}{b}$ and $\frac{b}{a}$—associated with any ratio $a : b$ and interpret them in context. Students practice finding unit rates and using them to solve various problems.

Students then use their understanding of ratios and rates to make sense of percentages. Just as a unit rate can be interpreted in context as a rate per 1, a percentage can be interpreted in context as a rate per 100.

Throughout the unit, students can use familiar representations such as tables and double number line diagrams in their reasoning. Sometimes a particular representation is suggested to help students make connections or to make sense of a situation. At other times students decide which representations to use, if needed.




| time (minutes) | percentage |
|---|---|
| 90 | 125 |
| 18 | 25 |
| 72 | 100 |

In a later unit, students will write equations of the form $px = q$ to represent situations where the value corresponding to 100% is unknown and will solve such equations. In grade 7, students will rely on their knowledge of equivalent ratios and unit rates to make sense of proportional relationships and constants of proportionality. Their understanding of percentages will support them in reasoning about percent increase and decrease.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, explaining, and justifying. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Interpret**

- Unit rates in different contexts (Lesson 5).
- A context in which identifying a unit rate is helpful (Lesson 8).
- Situations involving constant speed (Lesson 9).
- Diagrams used to represent percentages (Lessons 11 and 12).
- Situations involving measurement, rate, and cost (Lesson 17).

**Explain**

- Reasoning for estimating and sorting measurements (Lesson 1).
- Reasoning about relative sizes of units of measurement (Lesson 2).
- Reasoning for comparing rates (Lessons 4 and 7).
- Reasoning about percentages (Lesson 11).
- Strategies for finding missing information involving percentages (Lesson 14).

**Justify**

- Reasoning about equivalent ratios and unit rates (Lesson 6).
- Reasoning about finding percentages (Lessons 15 and 16).
- Reasoning about costs and time (Lesson 17).

In addition, students have opportunities to generalize about equivalent ratios, unit rates, and percentages from multiple contexts and with reference to benchmark percentages, tape diagrams, and other mathematical representations. Students can also be expected to describe measurements and observations, describe and compare situations involving percentages, compare speeds, compare prices, and critique reasoning about costs and time.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.3.2 | order | |
| 6.3.4 | (good / better / best) deal<br>rate per 1 | **unit price**<br>same speed |
| 6.3.5 | **unit rate** | |
| 6.3.6 | result | **unit rate** |
| 6.3.7 | | meters per second<br>(good / better / best) deal |
| 6.3.8 | | at this rate |
| 6.3.9 | **pace** | **speed** |
| 6.3.10 | **percentage**<br>___% of | |
| 6.3.11 | | tick marks |
| 6.3.12 | ___% as much as | ___% of |
| 6.3.14 | | |
| 6.3.15 | | **percentage** |

## Section A: Units of Measurement and Unit Conversion

- Lesson 1: Anchoring Units of Measurement
- Lesson 2: Measuring with Different-Size Units
- Lesson 3: Converting Units

## Section B: Rates

- Lesson 4: Comparing Speeds and Prices
- Lesson 5: Interpreting Rates
- Lesson 6: Equivalent Ratios Have the Same Unit Rates
- Lesson 7: More Rate Comparisons
- Lesson 8: Solving Rate Problems
- Lesson 9: More about Constant Speed

## Section C: Percentages

- Lesson 10: What Are Percentages?
- Lesson 11: Representing Percentages with Double Number Line Diagrams
- Lesson 12: Representing Percentages in Different Ways


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- Lesson 13: Benchmark Percentages
- Lesson 14: Solving Percentage Problems
- Lesson 15: Finding This Percent of That
- Lesson 16: Finding the Percentage

### Section D: Let's Put It to Work

- Lesson 17: Painting a Room

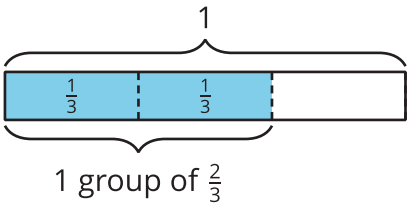
## Unit 4: Dividing Fractions

This unit develops students' understanding of division of fractions by fractions. This work draws on students' prior knowledge of multiplication, division, and the relationship between the two. It also builds on concepts from grades 3 to 5 about multiplicative situations—equal-size groups, multiplicative comparison, and the area of a rectangle—and about fractions.

Students begin by exploring meanings of division and the relationship between the quantities in division situations. They recall that we can think of dividing as finding an unknown factor in a multiplication equation. In situations involving equal-size groups, division can be used to answer two questions: "How many groups?" and "How much in each group?"

Next, students investigate ways to answer those two questions. They reason about situations in which the size of a group is known but the number of groups is not (as in, "How many $\frac{2}{3}$s are in 1?") and in which the number of groups is know but the size is not (as in, "What is in each bottle if there are 14 liters in $3\frac{1}{2}$ bottles?"). They also explore division in situations involving multiplicative comparison.

$$? \cdot \frac{2}{3} = 1 \qquad 1 \div \frac{2}{3} = ?$$




Students then apply their insights to generalize the process of finding quotients. In reasoning repeatedly to find the value of expressions such as $6 \div \frac{1}{4}$, $6 \div \frac{3}{4}$, and $6 \div \frac{a}{4}$, students notice regularity: Dividing a number by a fraction $\frac{a}{b}$ is the same as multiplying that number by $\frac{b}{a}$.

Students go on to use this algorithm to solve problems about geometric figures that have fractional length, area, or volume measurements. They also apply the concepts from the unit to solve multi-step problems involving fractions in other contexts.

Throughout the unit, students interpret and create equations and diagrams to make sense of the relationship between known and unknown quantities.

A deeper understanding of multiplication, division, and ways to represent them will support students in reasoning about decimal operations as well as in writing and solving variable equations later in the course.

*A note about diagrams:*

Because tape diagrams are a flexible tool for illustrating and reasoning about division of fractions, they are the primary representation used in this unit. Students may, however, create other representations to support their reasoning.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, representing, justifying, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are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using language to:

**Interpret and Represent**

- Situations involving division (Lessons 2, 3, 9, 12, and 16).
- Situations involving measurement constraints (Lesson 17).

**Justify**

- Reasoning about division and diagrams (Lessons 4 and 5).
- Strategies for dividing numbers (Lesson 11).
- Reasoning about volume (Lesson 15).

**Explain**

- How to create and make sense of division diagrams (Lesson 6).
- How to represent division situations (Lesson 9).
- How to find unknown lengths (Lesson 14).
- A plan for optimizing costs (Lesson 17).

In addition, students are expected to critique the reasoning of others about division situations and representations, and to make generalizations about division by comparing and connecting across division situations and across the representations used in reasoning about these situations. The *Lesson Syntheses* in Lessons 2 and 12 offer specific disciplinary language that may be especially helpful for supporting students in navigating the language of important ideas in this unit.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | **receptive** | **productive** |
| 6.4.1 | divisor<br>dividend | quotient |
| 6.4.2 | equation<br>interpretation<br>equal-size | How many groups of ___?<br>How many ___ in each group? |
| 6.4.3 | unknown | |
| 6.4.4 | whole | |
| 6.4.5 | | whole |
| 6.4.6 | | equal-size |
| 6.4.7 | times as ___<br>fraction of ___ | |
| 6.4.8 | container<br>section | unknown<br>fraction of ___ |
| 6.4.10 | observations | times as ___<br>numerator<br>denominator |
| 6.4.11 | **reciprocal** | |
| 6.4.13 | | gaps |
| 6.4.14 | packed | |
| 6.4.17 | assumption | packed |

## Section A: Making Sense of Division

- Lesson 1: Size of Divisor and Size of Quotient
- Lesson 2: Meanings of Division
- Lesson 3: Interpreting Division Situations

## Section B: Meanings of Fraction Division

- Lesson 4: How Many Groups? (Part 1)
- Lesson 5: How Many Groups? (Part 2)
- Lesson 6: Using Diagrams to Find the Number of Groups
- Lesson 7: What Fraction of a Group?
- Lesson 8: How Much in Each Group? (Part 1)
- Lesson 9: How Much in Each Group? (Part 2)


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### Section C: Algorithm for Fraction Division

- Lesson 10: Dividing by Unit and Non-Unit Fractions
- Lesson 11: Using an Algorithm to Divide Fractions

### Section D: Fractions in Lengths, Areas, and Volumes

- Lesson 12: Fractional Lengths
- Lesson 13: Rectangles with Fractional Side Lengths
- Lesson 14: Fractional Lengths in Triangles and Prisms
- Lesson 15: Volume of Prisms

### Section E: Let's Put It to Work

- Lesson 16: Solving Problems Involving Fractions
- Lesson 17: Fitting Boxes into Boxes

## Unit 5: Arithmetic in Base Ten

In this unit, students solidify their understanding of the base-ten number system, extend their use of the standard algorithms to add, subtract, and multiply decimals beyond tenths and hundredths, and learn to use algorithms to calculate quotients. The work here builds on what students learned in earlier grades about operations on whole numbers and decimals.

Students begin by exploring the use of decimals in a shopping context and by revisiting addition and subtraction of decimals, using both concrete representations and numerical calculations. The activities in the first section reinforce ideas about place value, properties of operations, the algorithms for adding and subtracting, and the relationship between addition and subtraction.

Next, students investigate various ways to find the product of two decimals: by using decimal fractions, writing equivalent expressions with whole numbers and unit decimals (such as 0.1 and 0.01), using diagrams and partial products, and reasoning about the relationship between a decimal and a related whole number. Students notice that the different methods of reasoning are governed by the same structure based on place value, which also underlies the standard algorithm for multiplication.

The next section focuses on division. Students have an opportunity to use base-ten blocks or diagrams to represent division of multi-digit numbers before exploring other numerical methods, such as using partial quotients and long division. Students progress through calculations of increasing complexity. They first divide whole numbers that give a whole-number quotient, and then divide whole numbers with a (terminating) decimal quotient. Next, they divide a decimal by a whole number, and finally a decimal by a decimal.

Mai's diagram for $62 \div 5$

Lin's calculation for $62 \div 5$


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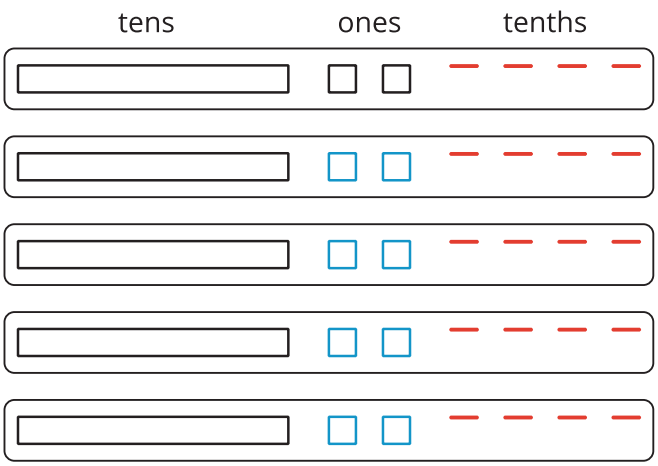


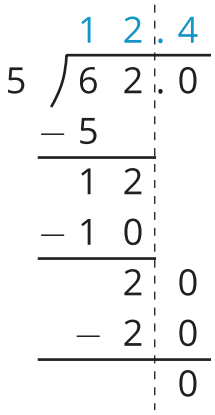


In the last section, students apply the mathematics from the unit to solve problems in applied situations. These require students to interpret quantities and results in context, and to consider appropriate levels of precision in their work.

*A note about materials:*

Base-ten blocks and paper versions of them will be useful throughout the unit. Consider preparing commercially produced base-ten blocks, if available, or printing representations of base-ten units on card stock, cutting them out, and organizing them for easy reuse.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as explaining, interpreting, and comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to**:**

**Explain**

- Processes of estimating and finding costs (Lesson 1).
- Approaches to adding and subtracting decimals (Lesson 4).
- Reasoning about products and quotients involving powers of 10 (Lesson 5).
- Methods for multiplying decimals (Lesson 8).
- Reasoning about relationships among measurements (Lesson 15).

**Interpret**

- Representations of decimals (Lesson 2).
- Base-ten diagrams showing addition or subtraction of decimals (Lesson 3).
- Area diagrams showing products of decimals (Lesson 7).
- Base-ten diagrams representing division of a whole number or a decimal by a whole number (Lessons 9, 12).
- Calculations showing partial quotients or steps in long division (Lessons 10, 11, 12).

**Compare**

- Base-ten diagrams with numerical calculations (Lesson 4).
- Methods for multiplying decimals (Lesson 6).
- Methods for finding quotients (Lessons 10, 11, 12).
- Measurements of two- and three-dimensional objects (Lesson 15).


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In addition, students are expected to describe decimal values to hundredths, generalize about multiplication by powers of 10 and about decimal measurements, critique approaches to operations on decimals, and justify strategies for finding sums, differences, products, and quotients.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.

| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.5.1 | digits<br>budget<br>at least | |
| 6.5.2 | base-ten diagram<br>compose<br>vertical calculation | place value<br>digits |
| 6.5.3 | decompose | |
| 6.5.4 | method | compose<br>decompose |
| 6.5.5 | powers of 10 | product<br>decimal point |
| 6.5.7 | partial products | method |
| 6.5.9 | remainder | |
| 6.5.10 | partial quotients | divisor |
| 6.5.11 | **long division** | |
| 6.5.12 | | remainder |
| 6.5.13 | | **long division** |
| 6.5.14 | precision<br>accuracy<br>operation | |

## Section A: Exploring, Adding, and Subtracting Decimals

- Lesson 1: Using Decimals in a Shopping Context
- Lesson 2: Using Diagrams to Represent Addition and Subtraction
- Lesson 3: Adding and Subtracting Decimals with Few Non-Zero Digits
- Lesson 4: Adding and Subtracting Decimals with Many Non-Zero Digits


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## Section B: Multiplying Decimals

- Lesson 5: Using Fractions to Multiply Decimals
- Lesson 6: Methods for Multiplying Decimals
- Lesson 7: Using Diagrams to Represent Multiplication
- Lesson 8: Calculating Products of Decimals

## Section C: Dividing Decimals

- Lesson 9: Using Base-Ten Diagrams to Divide
- Lesson 10: Using Partial Quotients
- Lesson 11: Using Long Division
- Lesson 12: Dividing Numbers that Result in a Decimal
- Lesson 13: Dividing a Decimal by a Decimal

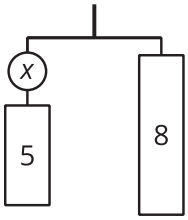
## Section D: Let's Put It to Work

- Lesson 14: Solving Problems Involving Decimals
- Lesson 15: Making and Measuring Boxes

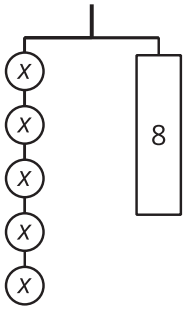
# Unit 6: Expressions and Equations

In this unit, students apply their understanding of arithmetic to reason about algebraic expressions and equations.

In the first section, students work with equations of the form $x + p = q$ and $px = q$ where $p$ and $q$ are positive rational numbers. They use tape diagrams and hanger diagrams to reason about the meaning of equations, and to develop an understanding that to solve an equation is to find a value that would make the equation true. Students end the section by identifying, interpreting, and writing equations to represent and solve real-world problems.




$x + 5 = 8$




$5x = 8$

In the second section, students write algebraic expressions and evaluate them for given values. They identify and write equivalent expressions, reasoning using diagrams, the distributive property, and other properties of operations.

The third section is all about exponents. Students write expressions with a whole-number exponent and a base that may be a whole number, a fraction, or a variable. They analyze such expressions for equivalence, as well as use the conventional order of operations to evaluate them. Students also identify solutions to simple exponential equations.

In the last two sections, students analyze real-world relationships between two quantities where one quantity depends on the other. They use tables, graphs, and equations to represent and reason about such relationships.

The work here prepares students to represent quantities and relationships involving all rational numbers in a later unit, as well as to solve equations that are more complex and work with proportional relationships in grade 7.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as interpreting, describing, and explaining. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to: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**Interpret**

- Tape diagrams involving letters that stand for numbers (Lesson 1).
- The parts of an equation (Lesson 5).
- Numerical expressions involving exponents (Lesson 12).
- Different representations of the same relationship between quantities (Lesson 17).

**Describe**

- Solutions to equations (Lesson 2).
- Stories represented by given equations (Lesson 5).
- Patterns of growth that can be represented using exponents (Lesson 12).
- Relationships between independent and dependent variables using tables, graphs, and equations (Lesson 16).

**Explain**

- The meaning of a solution using hanger diagrams (Lesson 3).
- How to solve an equation (Lesson 4).
- How to use equations to solve problems involving percentages (Lesson 6).
- How to determine whether two expressions are equivalent, including with reference to diagrams (Lesson 8).
- Strategies for determining whether expressions are equivalent (Lesson 14).

In addition, students are expected to compare descriptions of situations, expressions, equations, diagrams, tables, and graphs. They generalize about properties of operations and strategies for solving equations. Students also justify claims about equivalent expressions and justify reasoning when evaluating expressions.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.6.1 | value (of a variable) | operation |
| 6.6.2 | **variable**<br>**coefficient**<br>**solution to an equation**<br>true equation<br>false equation | value (of a variable) |
| 6.6.3 | each side (of an equal sign)<br>balanced hanger diagram | |
| 6.6.4 | solve (an equation) | each side (of an equal sign) |
| 6.6.6 | | true equation<br>false equation |
| 6.6.8 | **equivalent expressions**<br>commutative property | |
| 6.6.9 | distributive property<br>area as a product<br>area as a sum | |
| 6.6.10 | **term** | **equivalent expressions** |
| 6.6.12 | to the power | |
| 6.6.13 | | to the power<br>**exponent** |
| 6.6.15 | | **solution to an equation** |
| 6.6.16 | **independent variable**<br>**dependent variable**<br>horizontal axis<br>vertical axis | **variable**<br>relationship |
| 6.6.17 | coordinates | |
| 6.6.18 | plot | |

## Section A: Equations in One Variable

- Lesson 1: Tape Diagrams and Equations
- Lesson 2: Truth and Equations
- Lesson 3: Staying in Balance
- Lesson 4: Practice Solving Equations
- Lesson 5: Represent Situations with Equations


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- Lesson 6: Percentages and Equations

### Section B: Equal and Equivalent

- Lesson 7: Write Expressions with Variables
- Lesson 8: Equal and Equivalent
- Lesson 9: The Distributive Property, Part 1
- Lesson 10: The Distributive Property, Part 2
- Lesson 11: The Distributive Property, Part 3

### Section C: Expressions with Exponents

- Lesson 12: Meaning of Exponents
- Lesson 13: Expressions with Exponents
- Lesson 14: Evaluating Expressions with Exponents
- Lesson 15: Equivalent Exponential Expressions

### Section D: Relationships Between Quantities

- Lesson 16: Two Related Quantities, Part 1
- Lesson 17: Two Related Quantities, Part 2
- Lesson 18: More Relationships

### Section E: Let's Put It to Work

- Lesson 19: Tables, Equations, and Graphs, Oh My!

## Unit 7: Rational Numbers

In this unit, students learn about negative numbers and ways to represent them on a number line and the coordinate plane. They write and graph simple inequalities in one variable and determine the greatest common factor and least common multiple of two whole numbers. In grade 7, students will perform arithmetic operations with signed numbers and write and solve more complex inequalities.

Students begin by considering situations involving temperature or elevation and interpreting what negative numbers mean in those contexts. They also plot points to represent positive and negative values and their opposites. Previously, when students worked only with nonnegative numbers, magnitude and order were indistinguishable. In this unit, when comparing two signed numbers, students learn to distinguish between the absolute value of a number (magnitude) and a number's relative position on the number line (order).


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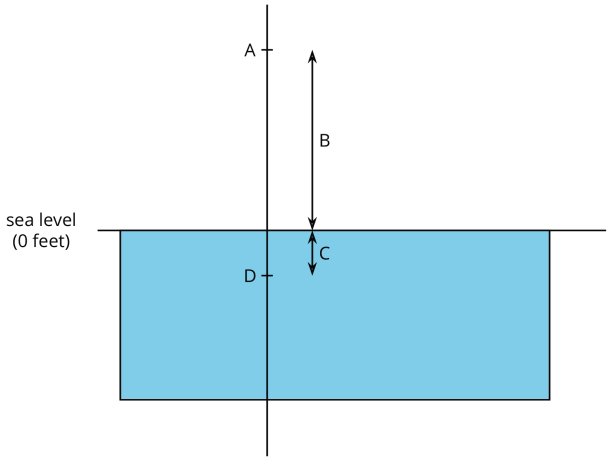


A. 15 feet

B. $|15|$ feet

C. $|-4|$ feet

D. -4 feet

Next, students use the symbols $<$ and $>$ to compare two values. They represent an unknown value with constraints as an inequality, and they graph the solutions on a number line. Students consider the inclusion or exclusion of boundary values and interpret solutions based on the context. In these grade 6 materials, inequality symbols in are limited to $<$ and $>$. However, in this unit students encounter situations that are best represented by both an inequality and an equation, such as "$x > 2$ or $x = 2$."

Then students use ordered pairs to describe pairs of numbers that include negative numbers. In grade 5, they plotted pairs of positive numbers on the coordinate grid. Here, they plot pairs of rational numbers in all four quadrants of the coordinate plane. They interpret the meanings of plotted points in given contexts and use coordinates to calculate horizontal or vertical distances between two points.

The last section of the unit returns to whole numbers. Students are introduced to common factors and common multiples. They determine the greatest common factor or the least common multiple of two numbers. They identify how these new concepts are involved in real-world situations and use their understanding to solve related problems.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as describing, interpreting, justifying, and generalizing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Describe and Interpret**

- Situations involving negative numbers (Lesson 1).
- Features of a number line (Lessons 2, 4 and 6).
- Situations involving elevation (Lesson 7).
- Situations involving minimums and maximums (Lesson 8).
- Points on a coordinate plane (Lessons 11 and 14).
- Situations involving factors and multiples (Lesson 18).

**Justify**

- Reasoning about magnitude (Lesson 3).
- Reasoning about a situation involving negative numbers (Lesson 5).
- Reasoning about solutions to inequalities (Lesson 9).
- That all possible pairs of factors have been identified (Lesson 16).

**Generalize**


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- The meaning of integers for a specific context (Lesson 5).
- Understanding of solutions to inequalities (Lesson 9).
- About the relationships between shapes (Lesson 10).
- About greatest common factors (Lesson 16).
- About least common multiples (Lesson 17).

In addition, students are expected to critique the reasoning of others, represent inequalities symbolically and in words, and explain how to order rational numbers and how to determine distances on the coordinate plane. Students also have opportunities to use language to compare magnitudes of positive and negative numbers, compare features of ordered pairs, and compare appropriate axes for different sets of coordinates.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.7.1 | **positive number**<br>**negative number**<br>temperature<br>degrees Celsius<br>elevation<br>sea level | number line<br>below zero |
| 6.7.2 | **opposite (numbers)**<br>**rational number**<br>location<br>distance (away) from zero | |
| 6.7.3 | **sign**<br>**inequality**<br>closer to 0<br>farther from 0 | greater than<br>less than |
| 6.7.4 | from least to greatest | temperature<br>elevation<br>sea level |
| 6.7.5 | positive change<br>negative change<br>context | |
| 6.7.6 | **absolute value** | **positive number**<br>**negative number**<br>distance (away) from zero |
| 6.7.7 | | closer to 0<br>farther from 0 |
| 6.7.8 | maximum<br>minimum | |
| 6.7.9 | requirement<br>**solution to an inequality** | |
| 6.7.10 | unbalanced hanger | inequality |
| 6.7.11 | **quadrant**<br>**coordinate plane**<br>$x$-coordinate<br>$y$-coordinate | |
| 6.7.12 | (line) segment | axis |
| 6.7.13 | degrees Fahrenheit | degrees Celsius |


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.7.14 | | **absolute value**<br>$x$-coordinate<br>$y$-coordinate |
| 6.7.16 | **common factor**<br>**greatest common factor (GCF)** | factor |
| 6.7.17 | **common multiple**<br>**least common multiple (LCM)** | multiple |

## Section A: Negative Numbers and Absolute Value

- Lesson 1: Positive and Negative Numbers
- Lesson 2: Points on the Number Line
- Lesson 3: Comparing Positive and Negative Numbers
- Lesson 4: Ordering Rational Numbers
- Lesson 5: Using Negative Numbers to Make Sense of Contexts
- Lesson 6: Absolute Value of Numbers
- Lesson 7: Comparing Numbers and Distance from Zero

## Section B: Inequalities

- Lesson 8: Writing and Graphing Inequalities
- Lesson 9: Solutions of Inequalities
- Lesson 10: Interpreting Inequalities

## Section C: The Coordinate Plane

- Lesson 11: Points in the Coordinate Plane
- Lesson 12: Constructing the Coordinate Plane
- Lesson 13: Interpreting Points in a Coordinate Plane
- Lesson 14: Distances in the Coordinate Plane
- Lesson 15: Shapes in the Coordinate Plane

## Section D: Common Factors and Common Multiples

- Lesson 16: Common Factors
- Lesson 17: Common Multiples
- Lesson 18: Using Common Multiples and Common Factors

## Section E: Let's Put It to Work

- Lesson 19: Drawing in the Coordinate Plane


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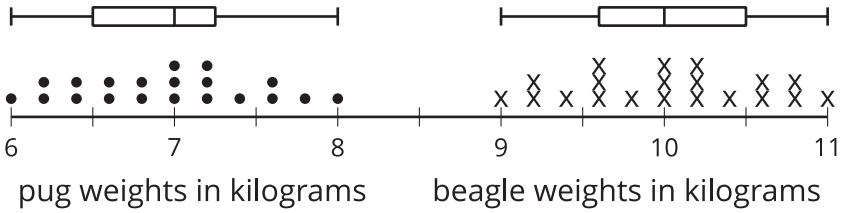
## Unit 8: Data Sets and Distributions

In this unit, students learn about populations and study variables associated with a population. They begin by classifying questions as either statistical or non-statistical—based on whether variable data is necessary to answer the question. This leads to further investigation into variability and data displays, such as dot plots and histograms. As students visualize data, they begin to describe the distribution of data more precisely as they work with mean and mean absolute deviation (MAD).

After working with those statistics, students begin to recognize that some distributions are not well-suited to description by mean and MAD. Students are introduced to median, range, and interquartile range as additional measures of center and variability that can be used to describe distributions in some situations. That also leads to the box plot as an additional way to visualize data.

Note that the introduction of mean absolute deviation is used as an introductory model for understanding variability. Although standard deviation is more mathematically useful, its calculation and meaning may be difficult for students at this level without an understanding of normal distributions. In later courses, when student understanding of variability and their exposure to additional distributions is expanded, students will learn about standard deviation and evolve their understanding away from mean absolute deviation.




**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as justifying, representing, and interpreting. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Justify**

- Reasoning for matching data sets to questions (Lesson 2).
- Reasoning about dot plots (Lesson 3).
- Reasoning about mean and median (Lesson 13).
- Reasoning about changes in mean and median (Lesson 14).
- Reasoning about which information is needed (Lesson 17).
- Which summaries and graphs best represent given data sets (Lesson 18).

**Represent**

- Data using dot plots (Lessons 3 and 4).
- Data using histograms (Lesson 7).
- Mean using bar graphs (Lesson 9).
- Data with five number summaries (Lesson 15).
- Data using box plots (Lesson 16).

**Interpret**

- Dot plots (Lessons 4 and 11).
- Histograms (Lessons 6 and 18).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- Mean of a data set (Lesson 9).
- Five-number summaries (Lesson 15).
- Box plots (Lesson 16).

In addition, students are expected to critique the reasoning of others, describe how quantities are measured, describe and compare features and distributions of data sets, generalize about means and distances in data sets, generalize categories for sorting data sets, and generalize about statistical questions. Students are also expected to use language to compare questions that produce numerical and categorical data, compare dot plots and histograms, and compare histograms and bar graphs.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.8.1 | **numerical data**<br>**categorical data**<br>**dot plot** | |
| 6.8.2 | **statistical question**<br>**variability** | |
| 6.8.3 | **distribution**<br>**frequency** | bar graph |
| 6.8.4 | typical | |
| 6.8.5 | **center**<br>**spread** | **variability** |
| 6.8.6 | **histogram**<br>bins | **distribution**<br>**center** |
| 6.8.7 | | **statistical question**<br>**spread** |
| 6.8.8 | symmetrical<br>peak<br>cluster<br>unusual value | **numerical data**<br>**categorical data**<br>gap |
| 6.8.9 | **average**<br>**mean**<br>fair share | |
| 6.8.10 | **measure of center**<br>balance point | |
| 6.8.11 | **mean absolute deviation (MAD)**<br>measure of spread | symmetrical<br>**mean** |
| 6.8.12 | | **mean absolute deviation (MAD)**<br>typical |
| 6.8.13 | **median** | **measure of center** |
| 6.8.14 | | peak<br>cluster<br>unusual value |
| 6.8.15 | **range**<br>**quartile**<br>**interquartile range (IQR)**<br>five-number summary | measure of spread<br>minimum<br>maximum |


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.8.16 | **box plot**<br>whisker | **median**<br>**interquartile range (IQR)** |
| 6.8.17 | | **range**<br>**quartile** |
| 6.8.18 | | **dot plot**<br>**histogram**<br>**box plot** |

## Section A: Data, Variability, and Statistical Questions

- Lesson 1: Got Data?
- Lesson 2: Statistical Questions

## Section B: Dot Plots and Histograms

- Lesson 3: Representing Data Graphically
- Lesson 4: Dot Plots
- Lesson 5: Using Dot Plots to Answer Statistical Questions
- Lesson 6: Interpreting Histograms
- Lesson 7: Using Histograms to Answer Statistical Questions
- Lesson 8: Describing Distributions on Histograms

## Section C: Measures of Center and Variability

- Lesson 9: Mean
- Lesson 10: Finding and Interpreting the Mean as the Balance Point
- Lesson 11: Variability and MAD
- Lesson 12: Using Mean and MAD to Make Comparisons

## Section D: Median and IQR

- Lesson 13: Median
- Lesson 14: Comparing Mean and Median
- Lesson 15: Quartiles and Interquartile Range
- Lesson 16: Box Plots
- Lesson 17: Using Box Plots

## Section E: Let's Put It to Work

- Lesson 18: Using Data to Solve Problems


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## Unit 9: Putting It All Together

This optional unit consists of eleven lessons. The first section has four lessons about exploring our world. These lessons are independent of each other and explore working with estimation and decimals or large numbers. These lessons should be taught after Unit 3. The second section has five lessons about different systems of voting. These lessons build on each other and should be completed in order after Unit 3 has been taught. The final section has two lessons making connections between algebraic and geometric representations of topics in earlier units. This section should be taught after Unit 8 and the lessons should be completed in order.

All related standards in this unit have been addressed in prior units. These sections provide an *optional* opportunity for students to go more deeply and make connections between domains.

**Progression of Disciplinary Language**

In this unit, teachers can anticipate students using language for mathematical purposes, such as critiquing, justifying, and "comparing. Throughout the unit, students will benefit from routines designed to grow robust disciplinary language, both for their own sense-making and for building shared understanding with peers. Teachers can formatively assess how students are using language in these ways, particularly when students are using language to:

**Critique**

- Reasoning about Fermi problems (Lesson 1).
- Claims about percentages (Lesson 5).
- Reasoning about the fairness of voting systems (Lessons 8 and 9).

**Justify**

- Reasoning about Fermi problems (Lesson 1).
- Reasoning about the fairness of voting systems (Lessons 6, 7, 8, and 9).

**Compare**

- Sources of energy (Lessons 2 and 3).
- Rectangles and fractions (Lesson 10).
- Voting systems (Lessons 6 and 7).

In addition, students are expected to interpret and represent characteristics of the world population, describe distributions of voters, and generalize about decomposition of area and numbers.

The table shows lessons where new terminology is first introduced in this course, including when students are expected to understand the word or phrase receptively and when students are expected to produce the word or phrase in their own speaking or writing. Terms that appear bolded are in the Glossary. Teachers should continue to support students' use of a new term in the lessons that follow where it was first introduced.


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| lesson | new terminology | |
|---|---|---|
| | receptive | productive |
| 6.9.2 | kilowatt-hour (kWh) | |
| 6.9.5 | in favor<br>majority | |
| 6.9.6 | plurality<br>runoff | majority |
| 6.9.8 | in all<br>fair | |
| 6.9.10 | mixed number | |

## Section A: Exploring Our World

- Lesson 1: Fermi Problems
- Lesson 2: Energy Flow
- Lesson 3: Making Paper
- Lesson 4: If Our Class Were the World

## Section B: Voting

- Lesson 5: How Do We Choose?
- Lesson 6: More than Two Choices
- Lesson 7: Comparing Voting Systems
- Lesson 8: Picking Representatives
- Lesson 9: Designing Districts

## Section C: Making Connections

- Lesson 10: Rectangle Madness
- Lesson 11: Rectangle Fractions


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## Pacing Guide

*Number of days includes assessments. Upper bound of range includes optional lessons.*

| | Grade 6 | Grade 7 | Grade 8 |
|---|---|---|---|
| week 1 | Unit 1<br>Area and Surface Area<br>(20–22 days) (MA)<br>Optional Lesson: 16, 19 | Unit 1<br>Scale Drawings<br>(12–15 days)<br>Optional Lessons: 6, 8, 13 | Unit 1<br>Rigid Transformations and Congruence<br>(20 days) (MA)<br>Optional Lessons: none |
| week 2 | | | |
| week 3 | | | |
| week 4 | | Unit 2<br>Introducing Proportional Relationships<br>(16–17 days)<br>Optional Lessons: 14 | |
| week 5 | Unit 2<br>Introducing Ratios<br>(19 days)<br>Optional Lessons: none | | Unit 2<br>Dilations, Similarity, and Introducing Slope<br>(15 days)<br>Optional Lessons: none |
| week 6 | | | |
| week 7 | | | |
| week 8 | | Unit 3<br>Measuring Circles<br>(11–13 days)<br>Optional Lessons: 5, 11 | Unit 3<br>Linear Relationships<br>(16–17 days)<br>Optional Lessons: 11 |
| week 9 | Unit 3<br>Unit Rates and Percentages<br>(16–19 days)<br>Optional Lesson: 2, 9, 16 | | |
| week 10 | | | |
| week 11 | | Unit 4<br>Proportional Relationships and Percentages<br>(17–18 days)<br>Optional Lesson: 15 | |
| week 12 | | | Unit 4<br>Linear Equations and Linear Systems<br>(18 days)<br>Optional Lessons: none |
| week 13 | Unit 4<br>Dividing Fractions<br>(16–20 days) (MA)<br>Optional Lessons: 3, 4, 9, 16 | | |
| week 14 | | | |
| week 15 | | Unit 5<br>Rational Number Arithmetic<br>(18–19 days)<br>Optional Lesson: 10 | |
| week 16 | | | Unit 5<br>Functions and Volume<br>(23–25 days) (MA)<br>Optional Lessons: 18, 22 |
| week 17 | Unit 5<br>Arithmetic in Base Ten<br>(15–18 days) (MA)<br>Optional Lessons: 2, 9, 15 | | |
| week 18 | | | |
| week 19 | | Unit 6<br>Expressions, Equations, and Inequalities<br>(25 days) (MA)<br>Optional Lessons: none | |
| week 20 | | | |
| week 21 | Unit 6<br>Expressions and Equations<br>(20–22 days) (MA)<br>Optional Lessons: 11, 18 | | Unit 6<br>Associations in Data<br>(12–13 days)<br>Optional Lesson: 11 |
| week 22 | | | |
| week 23 | | | |
| week 24 | | Unit 7<br>Angles, Triangles, and Prisms<br>(18–19 days)<br>Optional Lesson: 17 | Unit 7<br>Exponents and Scientific Notation<br>(18 days)<br>Optional Lessons: none |
| week 25 | Unit 7<br>Rational Numbers<br>(20–21 days)<br>Optional Lesson: 19 | | |
| week 26 | | | |
| week 27 | | | |
| week 28 | | Unit 8<br>Probability and Sampling<br>(21–23 days) (MA)<br>Optional Lessons: 17, 20 | Unit 8<br>Pythagorean Theorem and Irrational Numbers<br>(20 days)<br>Optional Lessons: none |
| week 29 | Unit 8<br>Data Sets and Distributions<br>(20–21 days) (MA)<br>Optional Lessons: 18 | | |
| week 30 | | | |
| week 31 | | | |
| week 32 | | Unit 9<br>Putting It All Together<br>(0–12 days)<br>Optional Lessons: all | Unit 9<br>Putting It All Together<br>(0–6 days)<br>Optional Lessons: all |
| week 33 | Unit 9<br>Putting It All Together<br>(0–11 days)<br>Optional Lessons: all | | |
| week 34 | | | |
| week 35 | | | |

(MA) = Unit has Mid-Unit Assessment

Total number of days for each course = Lessons + Assessments – Optional Lessons

Grade 6 = 146 Days  Grade 7= 138 Days  Grade 8 = 142 Days


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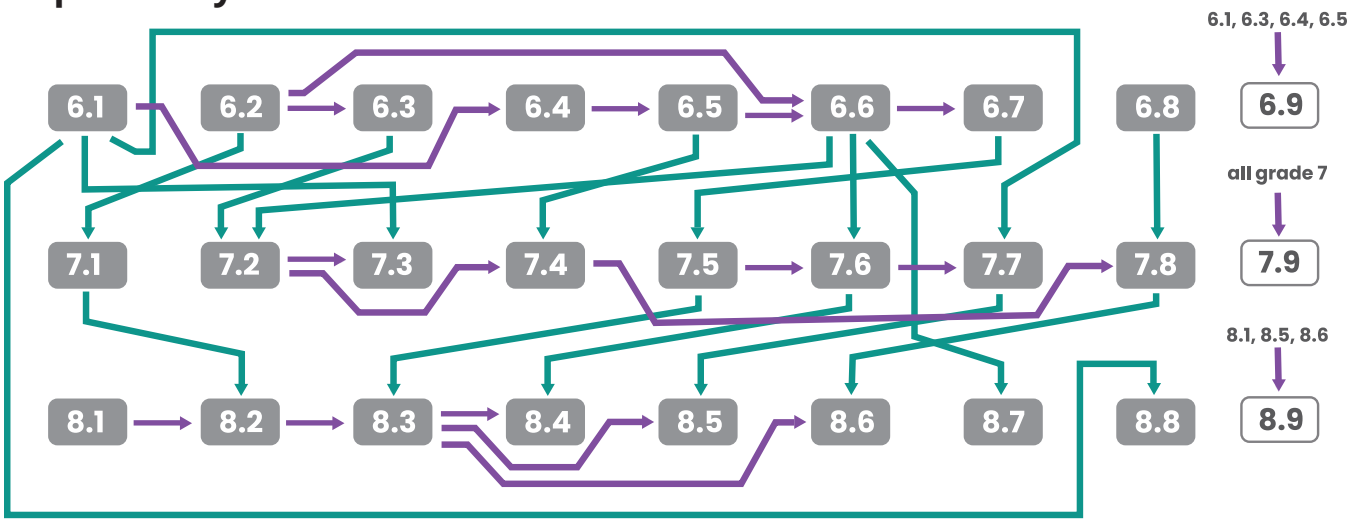
## Dependency Chart




In the unit dependency chart, an arrow indicates that a particular unit is designed for students who already know the material in a previous unit. Reversing the order of the units would have a negative effect on mathematical or pedagogical coherence. Examples:

- There is an arrow from 6.2 to 6.6. Students are expected to use their knowledge of contexts involving ratios (from 6.2) to write and solve equations representing such contexts (in 6.6).
- There is an arrow from 7.4 to 7.8. Students are expected to use their skills in representing percentages (from 7.4) when solving problems about probability (in 7.8).
- There is an arrow from 8.3 to 8.6. Students are expected to use their skills in writing and interpreting an equation that represents a line (from 8.3) to interpret the parameters in an equation that represents a line that fits a scatter plot (in 8.6).

The following chart shows unit dependencies across the curriculum for IM Grades 3–8.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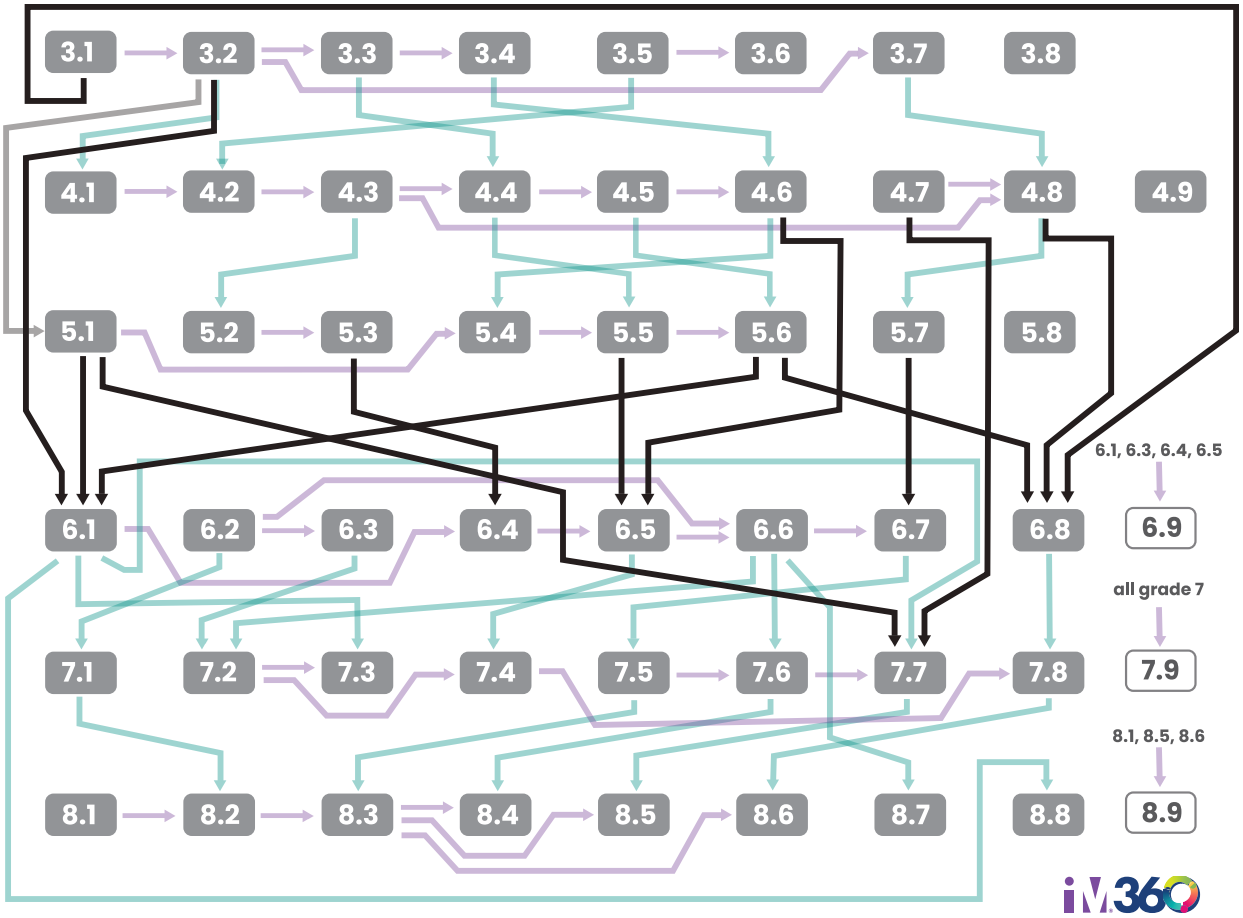
3.1
3.2
3.3
3.4
3.5
3.6
3.7
3.8
4.1
4.2
4.3
4.4
4.5
4.6
4.7
4.8
4.9
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
6.1
6.2
6.3
6.4
6.5
6.6
6.7
6.8
6.1, 6.3, 6.4, 6.5
6.9
all grade 7
7.1
7.2
7.3
7.4
7.5
7.6
7.7
7.8
7.9
8.1, 8.5, 8.6
8.1
8.2
8.3
8.4
8.5
8.6
8.7
8.8
8.9
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