Unit 7, Lesson 3

Writing and Solving Inequalities in One Variable

Let's solve problems by writing and solving inequalities in one variable.

3.1 Dinner for Drama Club

Kiran is getting dinner for his drama club on the evening of their final rehearsal. The budget for dinner is $60.

Kiran plans to buy some prepared dishes from a supermarket. The prepared dishes are sold by the pound, at $5.29 a pound. He also plans to buy two large bottles of sparkling water at $2.49 each.

1. Represent the constraints in the situation mathematically. If you use variables, specify what each one means.

2. How many pounds of prepared dishes can Kiran buy? Explain or show your reasoning.

3.2 Gasoline in the Tank

Han is about to mow some lawns in his neighborhood. His lawn mower has a 5-gallon fuel tank, but Han is not sure how much gasoline is in the tank.

He knows, however, that the lawn mower uses 0.4 gallon of gasoline per hour of mowing.

What are all the possible values for x, the number of hours Han can mow without refilling the lawn mower?

Write one or more inequalities to represent your response. Be prepared to explain or show your reasoning.

3.3 Different Ways of Solving

Andre and Priya used different strategies to solve the following inequality but reached the same solution.

$$2(2x + 1.5) < 18 - x$$

1. Make sense of each strategy until you can explain what each student has done.

   Andre

   $$\begin{aligned} 2(2x + 1.5) &= 18 - x \\ 4x + 3 &= 18 - x \\ 4x - 15 &= -x \\ -15 &= -5x \\ 3 &= x \end{aligned}$$

   Testing to see if $x = 4$ is a solution:

   $$\begin{aligned} 2(2 \cdot 4 + 1.5) &< 18 - 4 \\ 2(9.5) &< 14 \\ 19 &< 14 \end{aligned}$$

   The inequality is false, so 4 is not a solution. If a number greater than 3 is not a solution, the solution must be less than 3, or $3 > x$.

   Priya

   $$\begin{aligned} 2(2x + 1.5) &= 18 - x \\ 4x + 3 &= 18 - x \\ 5x + 3 &= 18 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$

   In $4x + 3 = 18 - x$, there is $4x$ on the left and $-x$ on the right.

   If x is a negative number, $4x + 3$ could be positive or negative, but $18 - x$ will always be positive.

   For $4x + 3 < 18 - x$ to be true, x must include negative numbers or x must be less than 3.

2. Here are four inequalities.

   a. $\frac{1}{5}p > -10$.

   b. $4(x + 7) \leq 4(2x + 8)$

   c. $-9n < 36$

   d. $\frac{c}{3} < -2(c - 7)$

   Work with a partner to decide on at least two inequalities to solve. Solve one inequality using Andre's strategy (by testing values on either side of the given solution), while your partner uses Priya's strategy (by reasoning about the parts of the inequality). Switch strategies for the other inequality.

Are you ready for more?

Using positive integers between 1 and 9 and each positive integer at most once, fill in values to get two constraints so that $x = 7$ is the only integer that will satisfy both constraints at the same time.

$\square x + \square < \square x + \square$

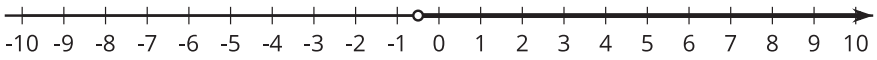
$\square x + \square > \square x + \square$

3.4 Matching Inequalities and Solutions

Match each inequality to a graph that represents its solutions. Be prepared to explain or show your reasoning.

1. $6x \leq 3x$
2. $\frac{1}{4}x > -\frac{1}{2}$
3. $5x + 4 \geq 7x$
4. $8x - 2 < -4(x - 1)$
5. $\frac{4x - 1}{3} > -1$
6. $\frac{12}{5} - \frac{x}{5} \leq x$

A



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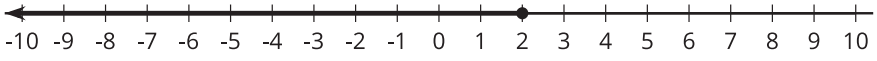

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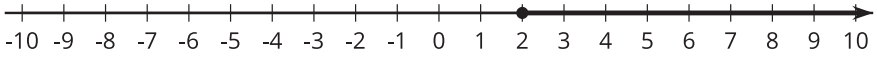

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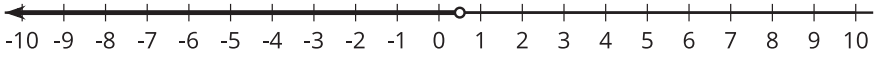

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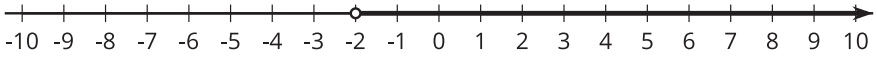

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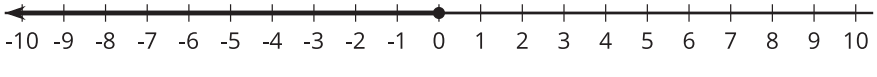

Lesson 3 Summary

Writing and solving inequalities can help us make sense of the constraints in a situation and solve problems. Let's look at an example.

Clare would like to buy a video game system that costs $130 and to have some extra money for games. She has saved $48 so far and plans on saving $5 of her allowance each week. How many weeks, w, will it be until she has enough money to buy the system and have some extra money remaining? To represent the constraints, we can write $48 + 5w \geq 130$. Let's reason about the solutions:

- Because Clare has $48 already and needs to have at least $130 to afford the game system, she needs to save at least $82 more.
- If she saves $5 each week, it will take at least $\frac{82}{5}$ weeks to reach $82.
- $\frac{82}{5}$ is 16.4. Any time shorter than 16.4 weeks won't allow her to save enough.
- Assuming she saves $5 at the end of each week (instead of saving smaller amounts throughout a week), it will be at least 17 weeks before she can afford the game system.

We can also solve by writing and solving a related equation to find the boundary value for w, and then determine whether the solutions are less than or greater than that value.

$$\begin{aligned} 48 + 5w &= 130 \\ 5w &= 82 \\ w &= \frac{82}{5} \\ w &= 16.4 \end{aligned}$$

- Substituting 16.4 for w in the original inequality gives a true statement. (When $w = 16.4$, we get $130 \geq 130$.)
- Substituting a value greater than 16.4 for w also gives a true statement. (When $w = 17$, we get $133 \geq 130$.)
- Substituting a value less than 16.4 for w gives a false statement. (When $w = 16$, we get $128 \geq 130$.)
- The solution set is therefore $w \geq 16.4$.

Sometimes the structure of an inequality can help us see whether the solutions are less than or greater than a boundary value. For example, to find the solutions to $3x > 8x$, we can solve the equation $3x = 8x$, which gives us $x = 0$. Then, instead of testing values on either side of 0, we could reason as follows about the inequality:

- If x is a positive value, then $3x$ would be less than $8x$.
- For $3x$ to be *greater* than $8x$, x must include negative values.
- For the solutions to include negative values, they must be less than 0, so the solution set would be $x < 0$.