# Rewriting Quadratic Expressions in Factored Form (Part 3)

Let’s look closely at some special kinds of factors.

## 8.1 Math Talk: Products of Large-ish Numbers

Evaluate mentally.

- $9 \cdot 11$
- $19 \cdot 21$
- $99 \cdot 101$
- $119 \cdot 121$

# 8.2 Can Products Be Written as Differences?

1. Clare claims that $(10 + 3)(10 - 3)$ is equivalent to $10^2 - 3^2$ and $(20 + 1)(20 - 1)$ is equivalent to $20^2 - 1^2$. Do you agree? Show your reasoning.

2. a. Use your observations from the first question to evaluate $(100 + 5)(100 - 5)$. Show your reasoning.

   b. Check your answer by computing $105 \cdot 95$.

3. Is $(x + 4)(x - 4)$ equivalent to $x^2 - 4^2$? Support your answer:

   With a diagram:

   Without a diagram:

| | $x$ | $4$ |
|---|---|---|
| $x$ | | |
| -4 | | |

4. Is $(x + 4)^2$ equivalent to $x^2 + 4^2$? Support your answer, either with or without a diagram.

## Are you ready for more?

1. Explain how your work in the previous questions can help you mentally evaluate $22 \cdot 18$ and $45 \cdot 35$.

2. Here is a shortcut that can be used to mentally square any two-digit number. Let's take $83^2$, for example.

   - 83 is $80 + 3$.
   - Compute $80^2$ and $3^2$, which give 6,400 and 9. Add these values to get 6,409.
   - Compute $80 \cdot 3$, which is 240. Double it to get 480.
   - Add 6,409 and 480 to get 6,889.

   Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

## 8.3 What If There Is No Linear Term?

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram.
(Heads up: One of them is impossible.)

| factored form | standard form |
|---|---|
| $(x - 10)(x + 10)$ | |
| $(2x + 1)(2x - 1)$ | |
| $(4 - x)(4 + x)$ | |
| | $x^2 - 81$ |
| | $49 - y^2$ |
| | $9z^2 - 16$ |
| | $25t^2 - 81$ |
| $(c + \frac{2}{5})(c - \frac{2}{5})$ | |
| | $\frac{49}{16} - d^2$ |
| $(x + \sqrt{5})(x - \sqrt{5})$ | |
| | $x^2 - 6$ |
| | $x^2 + 100$ |

## Lesson 8 Summary

Sometimes expressions in standard form don’t have a linear term. Can they still be written in factored form?

Let’s take $x^2 - 9$ as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0: $x^2 + 0x - 9$.

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is $(x + 3)(x - 3)$.

To check that this expression is indeed equivalent to $x^2 - 9$, we can expand the factored expression by applying the distributive property: $(x + 3)(x - 3) = x^2 - 3x + 3x + (-9)$. Adding $-3x$ and $3x$ gives 0, so the expanded expression is $x^2 - 9$.

In general, a quadratic expression that is a difference of two squares and has the form $a^2 - b^2$ can be rewritten as $(a + b)(a - b)$.

Here is a more complicated example: $49 - 16y^2$. This expression can be written as $7^2 - (4y)^2$, so an equivalent expression in factored form is $(7 + 4y)(7 - 4y)$.

What about $x^2 + 9$? Can it be written in factored form?

Let’s think about this expression as $x^2 + 0x + 9$. Can we find two numbers that multiply to make 9 and add up to 0? Here are factors of 9 and their sums:

- 9 and 1, sum: 10
- -9 and -1, sum: -10
- 3 and 3, sum: 6
- -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write $x^2 + 9$ in factored form using the kinds of numbers that we know about.