A Towering Sequence

Let's explore the Tower of Hanoi.

1.1 Which Three Go Together: What's Next?

Which three go together? Why do they go together?

A

2, 3, 5, 9, 17

B

2, 3, 4.5, 6.75

C

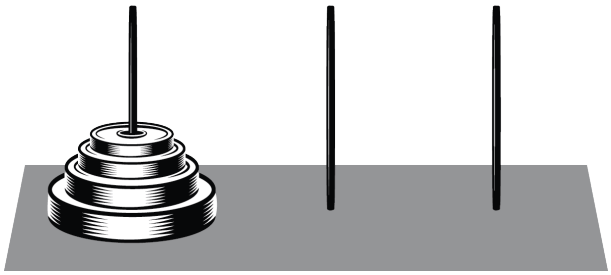
2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$

D

2, -4, 8, -16, 32

1.2 The Tower of Hanoi

In the Tower of Hanoi (huh-NOY) puzzle, a set of discs sits on 1 peg, and there are 2 other empty pegs.



A *move* in the Tower of Hanoi puzzle involves taking a disc and moving it to another peg. There are two rules:

- Move only 1 disc at a time.
- Never put a larger disc on top of a smaller one.

You complete the puzzle by building the complete tower on any peg other than the starting peg.

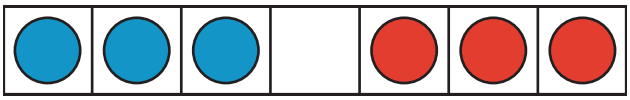
1. Using 3 discs, complete the puzzle. What is the smallest number of moves you can find?
2. Using 4 discs, complete the puzzle. What is the smallest number of moves you can find?
3. Jada says she used the solution for 3 discs to help her solve the puzzle for 4 discs. Describe how this might happen.

Are you ready for more?

What if a Tower of Hanoi puzzle with 64 discs was being solved at a rate of 1 move per second. How long would it take to solve this puzzle? Explain how you know.

1.3 Checker Jumping Puzzle

Some checkers are lined up. Blue checkers are on one side, red are on the other, and there is 1 empty space between them. A *move* in this checker game pushes any checker forward 1 space or jumps over any 1 checker of the other color. Jumping the same color is not allowed, moving backward is not allowed, and 2 checkers cannot occupy the same space.



The puzzle is completed when the colors are completely switched: ending up with blue on the right, red on the left, and 1 empty space between them.

1. Using 1 checker on each side, complete the puzzle. What is the smallest number of moves needed?
2. Using 3 checkers on each side, complete the puzzle. What is the smallest number of moves needed?
3. Estimate the number of moves needed if there are 4 checkers on each side, then test your guess.
4. Noah says he used the solution for 3 checkers on each side to help him solve the puzzle for 4 checkers. Describe how this might happen.

Lesson 1 Summary

A list of numbers, like 3, 5, 7, 9, 11, . . . or 1, 5, 13, 29, 61, . . . , is called a **sequence**.

There are many ways to define a sequence, but one way is to describe how each **term** relates to the term before it. For example, the sequence 3, 5, 7, 9, 11, . . . can be described this way: the starting term is 3, then each following term is 2 more than the term before it. The sequence 1, 5, 13, 29, 61, . . . can be described like this: the starting term is 1, then each following term is the sum of 3 and twice the previous term.

Throughout this unit, we will study several types of sequences along with ways to represent them.