



# Situations and Sequence Types

Let's decide what type of sequence we are looking at and how to represent it.

## 10.1 Describing Growth

1. Here is a geometric sequence: 16, 24, 36, 54, 81. What is the growth factor?
2. One way to describe its growth is to say it's growing by \_\_\_\_% each time. What number goes in the blank for the sequence 16, 24, 36, 54, 81? Be prepared to explain your reasoning.



## 10.2

## Finding Population Patterns

The table shows two animal populations growing over time.

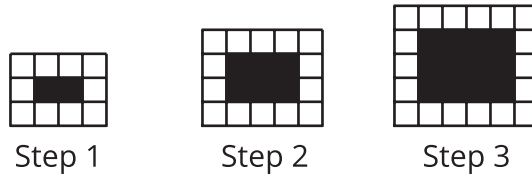
years since 1990	Population A	Population B
0	23,000	3,125
1	29,000	3,750
2	35,000	4,500
3	41,000	5,400
4	47,000	6,480

1. Are the sequences represented by Population A and Population B arithmetic or geometric? Explain how you know.
2. Write an equation to define Population A.
3. Write an equation to define Population B.
4. Does Population B ever overtake Population A? If so, when? Explain how you know.

## 10.3

## Finding Square Patterns

Define the sequence  $W$  so that  $W(n)$  is the number of white squares in Step  $n$ , and define the sequence  $B$  so that  $B(n)$  is the number of black squares in Step  $n$ .



1. Are the sequences  $W$  and  $B$  arithmetic, geometric, or neither? Explain how you know.
2. Write an equation for sequence  $W$ .
3. Write an equation for sequence  $B$ .



## Are you ready for more?

A definition for the  $n^{\text{th}}$  term of the Fibonacci sequence is surprisingly complicated. Humans have been interested in this sequence for a long time—it is named after an Italian mathematician who lived from around 1175 to 1250. The first person known to have stated the  $n^{\text{th}}$  term definition, though, was Abraham de Moivre, a French mathematician who lived from 1667 to 1754. So, this definition was unknown for hundreds of years! Here it is:

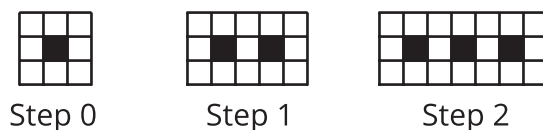
$$F(n) = \frac{1}{\sqrt{5}} \left( \left( \frac{1+\sqrt{5}}{2} \right)^n - \left( \frac{1-\sqrt{5}}{2} \right)^n \right)$$

1. Which form (recursive or  $n^{\text{th}}$  term) is more convenient to use for finding  $F(5)$ ? What about  $F(10)$ ?  $F(100)$ ?
2. What are some advantages and disadvantages of each form?

## Lesson 10 Summary

Some situations can be accurately modeled with geometric sequences, arithmetic sequences, or sequences that are neither geometric nor arithmetic.

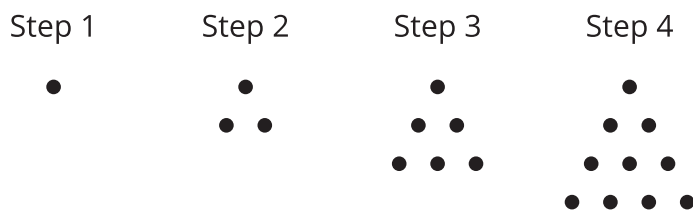
For example, here is a pattern of black squares surrounded by white squares, growing in steps.



The number of white squares in each step grows (8, 13, 18, . . .), with 5 more white squares each time. Since the same number of squares is added each time, the number of white squares forms an arithmetic sequence. The definition for the  $n^{\text{th}}$  term of  $W$ , where  $W(n)$  is the number of white squares in step  $n$ , is  $W(n) = 5n + 8$  for  $n \geq 0$ .

Geometric sequences are involved in situations such as population growth and scaling. For example, the sequence of areas we got when we imagined cutting a piece of paper in half at each step  $n$  in an earlier lesson.

Many situations lead to sequences that are neither geometric nor arithmetic. For example, consider this pattern of dots in which a new row of  $n$  dots introduced in each step:



The number of dots in each step grows (1, 3, 6, 10, . . .), but there is no constant being multiplied or added to get from term to term. If we create a graph of this sequence showing the number of dots as a function of the step number, the dots would form neither a linear nor an exponential shape. This sequence is neither geometric nor arithmetic, but it does have a pattern that we can define with an equation.