Unit 6, Lesson 9

# Scaling the Inputs

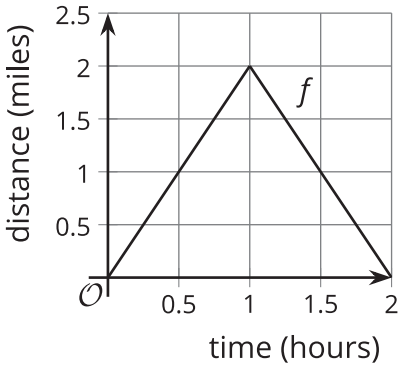
Let's use scale factors in different ways.

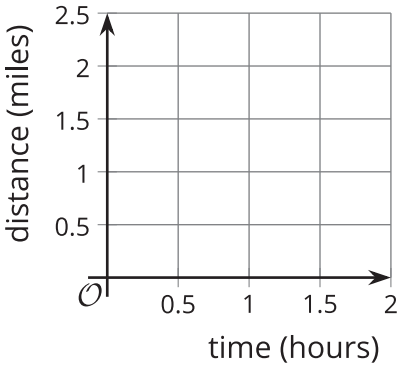
## 9.1 Out and Back

Every weekend, Elena takes a walk along the straight road in front of her house for 2 miles, then turns around and comes back home. Let's assume Elena walks at a constant speed.



Here is a graph of the function $f$ that gives her distance $f(t)$, in miles, from home as a function of time $t$ if she walks 2 miles per hour.




1. Sketch a graph of the function $g$ that gives her distance $g(t)$, in miles, from home as a function of time $t$ if she walks 4 miles per hour.
2. Write an equation for $g$ in terms of $f$. Be prepared to explain why your equation makes sense.

# 9.2 A New Set of Wheels

Remember Clare on the Ferris wheel? In the table, we have the function $F$ which gives her height $F(t)$ above the ground, in feet, $t$ seconds after starting her descent from the top. Today Clare tried out two new Ferris wheels.

- The first wheel is twice the height of $F$ and rotates at the same speed. The function $g$ gives Clare's height $g(t)$, in feet, $t$ seconds after starting her descent from the top.
- The second wheel is the same height as $F$ but rotates at half the speed. The function $h$ gives Clare's height $h(t)$, in feet, $t$ seconds after starting her descent from the top.

1. Complete the table for the function $g$.

| $t$ | $F(t)$ | $g(t)$ | $h(t)$ |
|---|---|---|---|
| 0 | 212 | | |
| 20 | 181 | | |
| 40 | 106 | | |
| 60 | 31 | | |
| 80 | 0 | | |

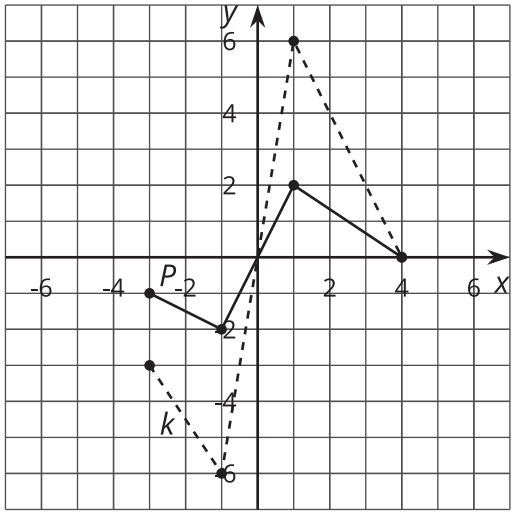
2. Explain why there is not enough information to find the exact values for $h(20)$ and $h(60)$.

3. Complete as much of the table as you can for the function $h$, modeling Claire's height on the second Ferris wheel.

4. Express $g$ and $h$ in terms of $F$. Be prepared to explain your reasoning.

# 9.3 The Many Transformations of a Function $P$

Function $k$ is a transformation of function $P$ due to a scale factor.




1. Write an equation for $k$ in terms of $P$.

2. On the same axes, graph the function $m$ where $m(x) = P(0.75x)$.
3. The highest point on the graph of $P$ is $(1, 2)$. What is the highest point on the graph of a function $n$ where $n(x) = P(5x)$? Explain or show your reasoning.

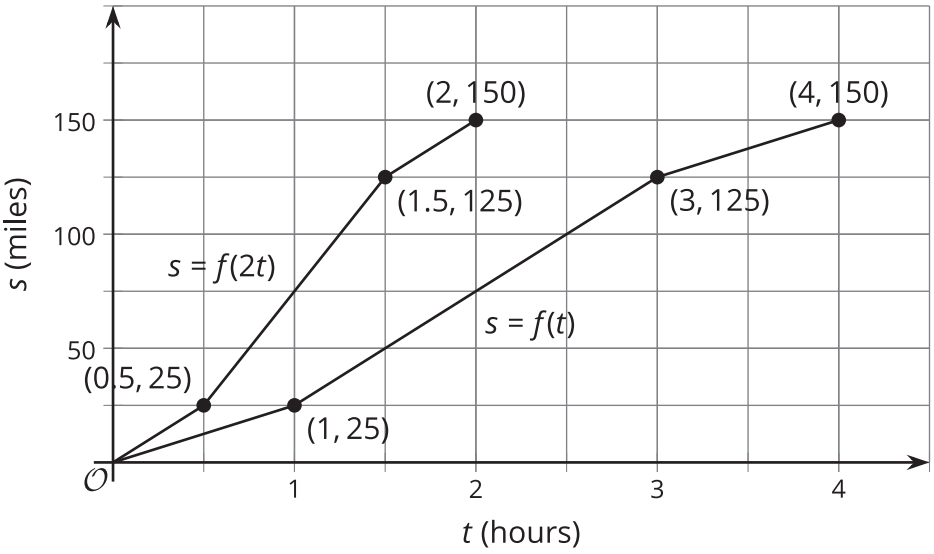
4. The point furthest to the right on the graph of $P$ is $(4, 0)$. If the point furthest to the right on the graph of a function $q$ is $(18, 0)$, write a possible equation for $q$ in terms of $P$.

### Are you ready for more?

What transformation takes $f(x) = 2x(x - 4)$ to $g(x) = 8x(x - 2)$?

## Lesson 9 Summary

Here are two graphs showing the distance traveled by two trains $t$ hours into their journeys. What do you notice?




Train A traveled 25 miles in 1 hour, and Train B traveled 25 miles in half the time. Similarly, Train A traveled 150 miles in 4 hours, while Train B traveled 150 miles in only 2 hours. Train B is traveling twice the speed of Train A.

A train traveling twice the speed gets to any particular point along the track in half the time, so the graph for Train B is compressed horizontally by a factor of $\frac{1}{2}$ when compared to the graph of Train A. If the function $f(t)$ represents the distance Train A travels in $t$ hours, then $f(2t)$ represents the distance Train B travels in $t$ hours, because Train B goes as far in $t$ hours as Train A goes in $2t$ hours.

If a different Train C were going one fourth the speed of Train A, then its motion would be represented by $s = f(0.25t)$ and the graph would be stretched horizontally by a factor of 4 since it would take four times as long to travel the same distance.