Unit 1, Lesson 9

What’s the Equation?

Let’s define sequences.

9.1 Math Talk: Multiplying Fractions

For the function $f(x) = 32 \cdot \left(\frac{3}{4}\right)^x$, evaluate mentally:

- $f(0)$
- $f(1)$
- $f(2)$
- $f(3)$

9.2 Filling Up

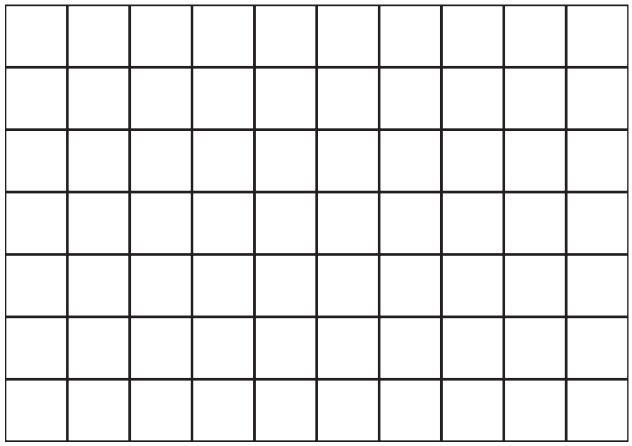
A full water cooler that is used to refill water bottles is in a room. The first person who comes in takes $\frac{1}{3}$ of the water to fill their bottle. Then the second person takes $\frac{1}{3}$ of what is left to fill their bottle. Then a third person takes $\frac{1}{3}$ of what is left. This pattern continues.

1. Complete the table for $C(n)$, the fraction of the original amount of water left after n people add to their bottle.
2. Write two definitions for C: one recursive and one non-recursive.
3. What is a reasonable domain for this function if the water cooler started with 1,280 ounces of water? Be prepared to explain your reasoning.

n	$C(n)$
0	
1	$\frac{2}{3}$
2	
3	
4	

9.3 Fibonacci Squares

1. On the grid, draw a square of side length 1 in the lower left corner. Draw another square of side length 1 that shares a side directly above the first square. Next, add a 2-by-2 square, with one side along the sides of both of the first two squares. Next, add a square with one side that goes along the sides of the previous two squares you created. Then do it again.



   Pause here for your teacher to check your work.

2. Write a sequence that lists the side lengths of the squares you drew.

3. Predict the next two terms in the sequence.

4. Describe how each square’s side length depends on previous side lengths.

5. Let $F(n)$ be the side length of the n^{th} square. So $F(1) = 1$ and $F(2) = 1$. Write a recursive definition for F.

Are you ready for more?

1. Is the Fibonacci sequence arithmetic, geometric, or neither? Explain how you know.

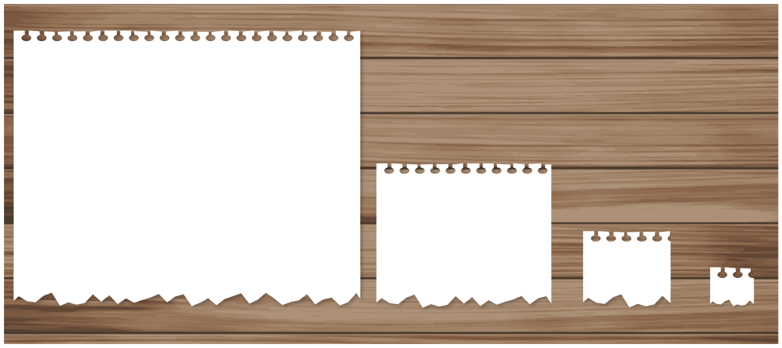
2. Look at quotients $\frac{F(2)}{F(1)}, \frac{F(3)}{F(2)}, \frac{F(4)}{F(3)}, \frac{F(5)}{F(4)}, \frac{F(6)}{F(5)}$. What do you notice about this sequence of numbers?

3. The 15th through 19th Fibonacci numbers are 610, 987, 1597, 2584, 4181. What do you notice about the quotients $\frac{F(16)}{F(15)}, \frac{F(17)}{F(16)}, \frac{F(18)}{F(17)}, \frac{F(19)}{F(18)}$?

Lesson 9 Summary

The model we use for a function can depend on what we want to do.

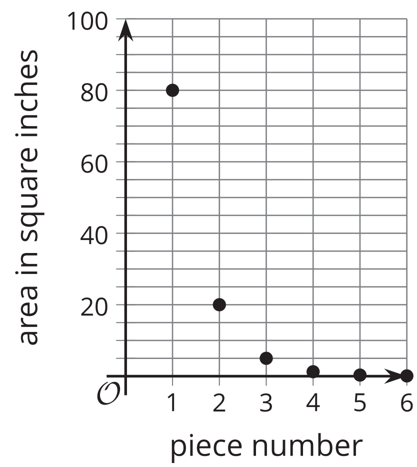
For example, an 8-by-10 piece of paper has area 80 square inches. Picture a set of pieces of paper, each half the length and half the width of the previous piece.



Define the sequence A so that $A(n)$ is the area, in square inches, of the n^{th} piece. Each new area is $\frac{1}{4}$ the previous area, so we can define A recursively like this:

$$A(1) = 80, A(n) = \frac{1}{4} \cdot A(n-1) \text{ for } n \geq 2$$

But for n-values larger than 5 or 6, the model isn't realistic since cutting a sheet of paper accurately when it is less than $\frac{1}{50}$ of a square inch isn't something we can do well with a pair of scissors. We can see this by looking at the graph of $y = A(n)$ shown here.



If we wanted to define the n^{th} term of A, it's helpful to first notice that the area of the n^{th} piece is given by $80 \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \cdots \cdot \frac{1}{4}$, where there are $n-1$ factors of $\frac{1}{4}$. Then we can write this definition:

$$A(n) = 80 \cdot \left(\frac{1}{4}\right)^{n-1}, n \geq 1$$

We can use this definition to calculate a value of A without having to calculate all the values that came before it. But since there are fewer than 10 values that make sense for A, since we can't cut very tiny pieces using scissors, in this situation we could use the first definition we found to calculate different values of A.

