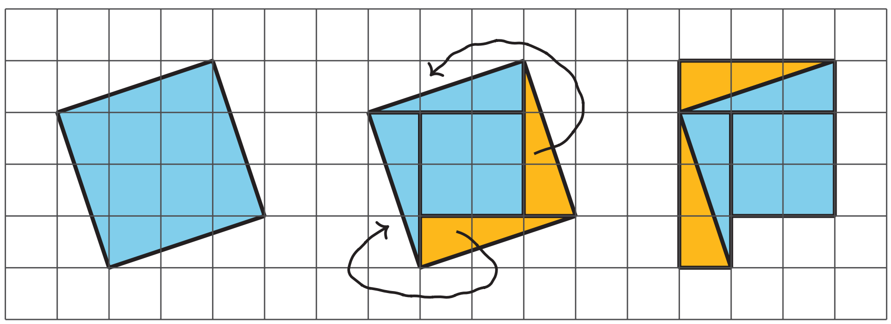
Unit 8 Family Support Materials

Pythagorean Theorem and Irrational Numbers

Section A: Side Lengths and Areas of Squares

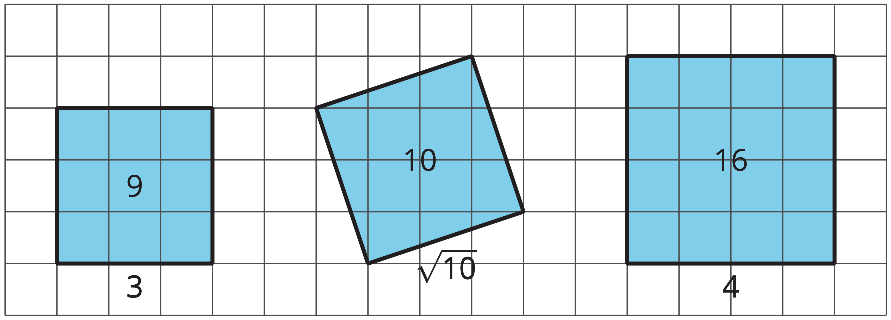
This week your student will be working with the relationship between the side length and area of squares. We know two main ways to find the area of a square:

- Multiply the square's side length by itself.
- Decompose and rearrange the square so that we can see how many square units are inside. For example, if we decompose and rearrange the tilted square in the diagram, we can see that its area is 10 square units.



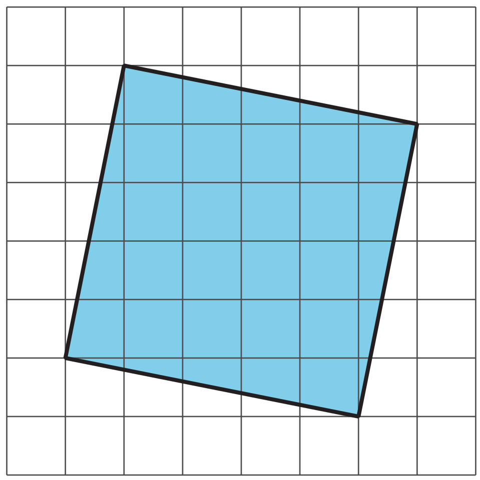
But what is the side length of this tilted square? It cannot be 3 units, since $3^2 = 9$, and it cannot be 4 units, since $4^2 = 16$. In order to write "the side length of a square whose area is 10 square units," we use a notation called a **square root**. We write "the square root of 10" as $\sqrt{10}$ and it means "the length of a side of a square whose area is 10 square units." All of these statements are true:

- $\sqrt{9} = 3$ because $3^2 = 9$.
- $\sqrt{16} = 4$ because $4^2 = 16$.
- $\sqrt{10}$ is the side length of a square whose area is 10 square units, and $\left(\sqrt{10}\right)^2 = 10$.



Here is a task to try with your student:

If each grid square represents 1 square unit, what is the side length of this titled square? Explain your reasoning.



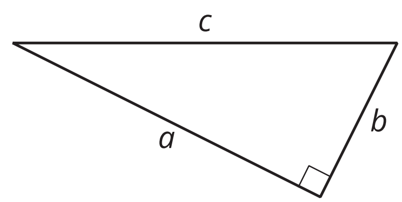
Solution:

The side length is $\sqrt{26}$ because the area of the square is 26 square units and the square root of the area of a square is the side length.

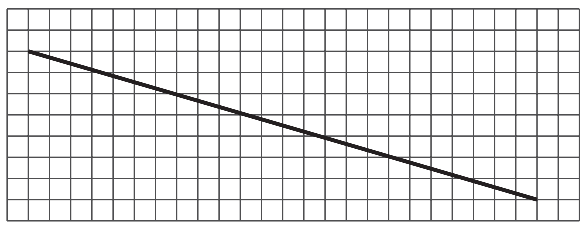
Section B: The Pythagorean Theorem

This week your student will work with the **Pythagorean Theorem**, which describes the relationship between the sides of any right triangle. A right triangle is any triangle with a right angle. The side opposite the right angle is called the **hypotenuse**, and the two other sides are called the **legs**.

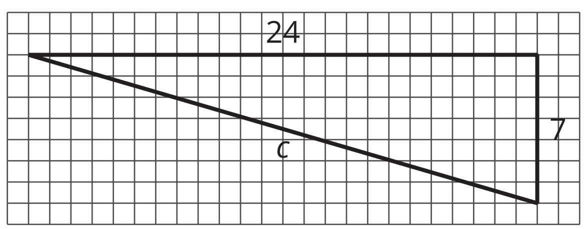
Here we have a triangle with hypotenuse c and legs a and b. The Pythagorean Theorem states that for any right triangle, the sum of the squares of the legs are equal to the square of the hypotenuse. In other words, $a^2 + b^2 = c^2$.



We can use the Pythagorean Theorem to tell if a triangle is a right triangle or not, to find the value of one side length of a right triangle if we know the other two, and to answer questions about situations that can be modeled with right triangles. For example, let's say we wanted to find the length of this line segment:



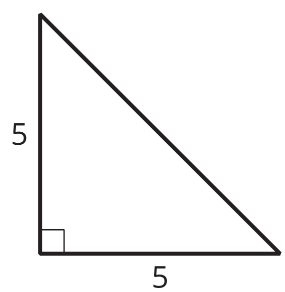
We can first draw a right triangle and determine the lengths of the two legs:



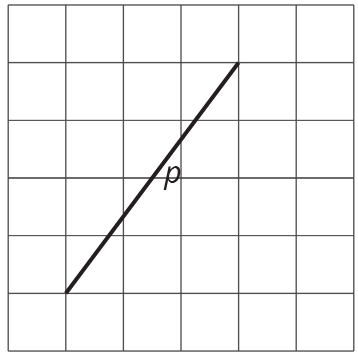
Next, since this is a right triangle, we know that $24^2 + 7^2 = c^2$, which means the length of the line segment is 25 units.

Here is a task to try with your student:

1. Find the length of the hypotenuse as an exact answer using a square root.



2. What is the length of line segment p? Explain or show your reasoning. (Each grid square represents 1 square unit.)



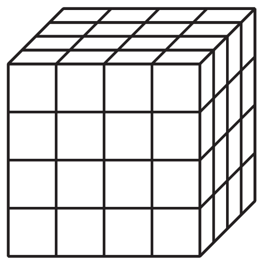
Solution:

1. The length of the hypotenuse is $\sqrt{50}$ units. With legs a and b both equal to 5 and an unknown value for the hypotenuse, c, we know the relationship $5^2 + 5^2 = c^2$ is true. That means $50 = c^2$, so c must be $\sqrt{50}$ units.
2. The length of p is $\sqrt{25}$, or 5, units. If we draw in the right triangle, we have legs of length 3 and 4 and hypotenuse p, so the relationship $3^2 + 4^2 = p^2$ is true. Since $3^2 + 4^2 = 25 = p^2$, we know that p must equal $\sqrt{25}$, or 5, units.

Section C: Side Lengths and Volumes of Cubes

This week your student will learn about both **cube roots** and decimal representations of rational and irrational numbers.

We previously learned that a square root is the side length of a square with a certain area. For example, if a square has an area of 16 square units, then its edge length is 4 units because $\sqrt{16} = 4$. Now, think about a solid cube. The cube has a volume, and the edge length of the cube is called the cube root of its volume. In this diagram, the cube has a volume of 64 cubic units:



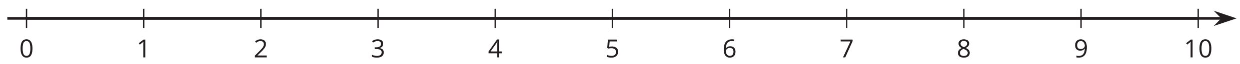
Even without the useful grid, using the volume, we can calculate that the edge length is 4 since $\sqrt[3]{64} = 4$.

Cube roots that are not integers are still numbers that we can plot on a number line. Consider the four numbers $\frac{1}{3}$, $\sqrt{40}$, $\sqrt[3]{30}$, and $\sqrt[3]{64}$. Since $\frac{1}{3}$ is rational, we can plot it by taking the interval between 0 and 1 and dividing it into 3 equal parts. Since $\sqrt{40}$ and $\sqrt[3]{30}$ are irrational, we can plot them on the number line by estimating what integers they are near.

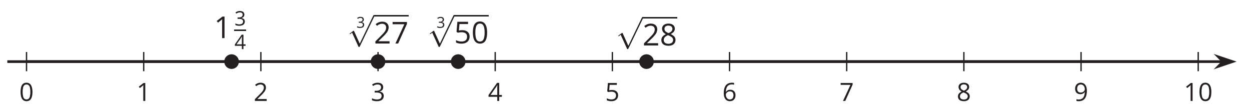
For example, $\sqrt{40}$ is between 6 and 7 since $\sqrt{40}$ is between $\sqrt{36}$ and $\sqrt{49}$. We know that $\sqrt{36} = 6$ and $\sqrt{49} = 7$. Similarly, $\sqrt[3]{30}$ is between 3 and 4 because $\sqrt[3]{30}$ is between $\sqrt[3]{27}$ and $\sqrt[3]{64}$. Our number line will look like this:

Here is a task to try with your student:

Plot the given numbers on the number line: $1\frac{3}{4}$, $\sqrt{28}$, $\sqrt[3]{27}$, $\sqrt[3]{50}$



Solution:



- For $1\frac{3}{4}$, we can subdivide the interval between 1 and 2 into 4 equal parts and find the 3rd part.
- Since $3^3 = 27$ means $\sqrt[3]{27} = 3$, we can plot $\sqrt[3]{27}$ at 3.
- $\sqrt[3]{50}$ is between 3 and 4 because 50 is between $3^3 = 27$ and $4^3 = 64$.
- $\sqrt{28}$ is between 5 and 6 because 28 is between $5^2 = 25$ and $6^2 = 36$.