Unit 5, Lesson 21

Cylinders, Cones, and Spheres

Let's find the volume of shapes.

21.1 Sphere Arguments

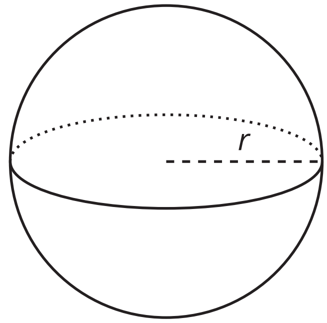
Four students each calculated the volume of a sphere with a radius of 9 centimeters, and they got four different answers.

- Han thinks it is 108 cubic centimeters.
- Jada gets 108π cubic centimeters.
- Tyler calculates 972 cubic centimeters.
- Mai says it is 972π cubic centimeters.

Do you agree with any of them? Explain your reasoning.

21.2 Sphere's Radius

The volume of this sphere with radius r is $V = 288\pi$.



This statement is true:

$$288\pi = \frac{4}{3}r^3\pi.$$

What is the value of r for this sphere? Explain how you know.

21.3 Info Gap: Unknown Dimensions

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

If your teacher gives you the problem card:

1. Silently read your card, and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need. “Can you tell me ________?”
3. Explain to your partner how you are using the information to solve the problem. “I need to know ________ because . . . .”

   Continue to ask questions until you have enough information to solve the problem.
4. Once you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.

If your teacher gives you the data card:

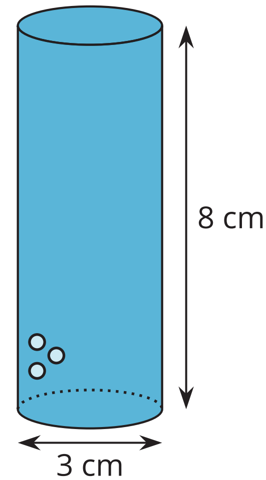
1. Silently read your card. Wait for your partner to ask for information.
2. Before telling your partner any information, ask, “Why do you need to know ________?”
3. Listen to your partner’s reasoning and ask clarifying questions. Only give information that is on your card. Do not figure out anything for your partner!

   These steps may be repeated.
4. Once your partner says they have enough information to solve the problem, read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

21.4 The Right Fit

A cylinder with diameter 3 centimeters and height 8 centimeters is filled with water.

Decide which figures described here, if any, could hold all of the water from the cylinder. Explain your reasoning.



1. Cone with a height of 8 centimeters and a radius of 3 centimeters.

2. Cylinder with a diameter of 6 centimeters and height of 2 centimeters.

3. Rectangular prism with a length of 3 centimeters, width of 4 centimeters, and height of 8 centimeters.

4. Sphere with a radius of 2 centimeters.

Are you ready for more?

A thirsty crow wants to raise the level of water in a cylindrical container so that it can reach the water with its beak.

- The container has a diameter of 2 inches and a height of 9 inches.
- The water level is currently at 6 inches.
- The crow can reach the water if it is 1 inch from the top of the container.

In order to raise the water level, the crow puts spherical pebbles in the container. If the pebbles are approximately $\frac{1}{2}$ of an inch in diameter, what is the fewest number of pebbles the crow needs to drop into the container in order to reach the water?

Lesson 21 Summary

The formula $V = \frac{4}{3}\pi r^3$ gives the volume of a sphere with radius r.

We can use the formula to find the volume of a sphere with a known radius. For example, if the radius of a sphere is 6 units, then the volume would be $\frac{4}{3}\pi(6)^3 = 288\pi$, or approximately 905 cubic units.

We can also use the formula to find the radius of a sphere if we only know its volume. For example, if we know that the volume of a sphere is 36π cubic units but we don't know the radius, then this equation is true:

$$36\pi = \frac{4}{3}\pi r^3$$

That means that $r^3 = 27$, so the radius r has to be 3 units in order for both sides of the equation to have the same value.

Many common objects, from water bottles to buildings to balloons, are similar in shape to rectangular prisms, cylinders, cones, or spheres—or even combinations of these shapes! Using the volume formulas for these shapes allows us to compare the volume of different types of objects, sometimes with surprising results.

For example, a cube-shaped box with side length 3 centimeters holds less than a sphere with radius 2 centimeters because the volume of the cube is 27 cubic centimeters ($3^3 = 27$) and the volume of the sphere is around 33.51 cubic centimeters ($\frac{4}{3}\pi \cdot 2^3 \approx 33.51$).