Using an Algorithm to Divide Fractions

Let's divide fractions using the rule we learned.

11.1 Math Talk: Multiplying Fractions

Find the value of each product mentally.

- $\frac{1}{8} \cdot 8$
- $\frac{1}{8} \cdot \frac{8}{3}$
- $\frac{9}{8} \cdot \frac{4}{3}$
- $1\frac{1}{8} \cdot \frac{4}{9}$

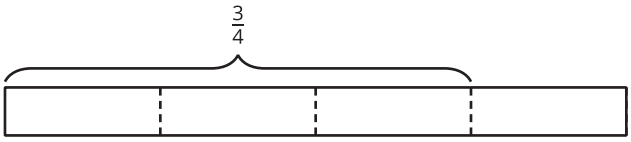
11.2 Dividing a Fraction by a Fraction

Work with a partner. One person works on the questions labeled "Partner A" and the other person works on those labeled "Partner B."

1. Partner A: Find the value of each expression by completing the diagram.

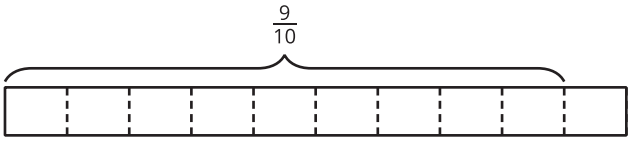
   a. $\frac{3}{4} \div \frac{1}{8}$

   How many $\frac{1}{8}$s are in $\frac{3}{4}$?



   b. $\frac{9}{10} \div \frac{3}{5}$

   How many $\frac{3}{5}$s are in $\frac{9}{10}$?



   Partner B:

   Elena said, "If I want to divide 4 by $\frac{2}{5}$, I can multiply 4 by 5 and then divide it by 2 or multiply it by $\frac{1}{2}$."

   Find the value of each expression using the strategy Elena described.

   a. $\frac{3}{4} \div \frac{1}{8}$

   b. $\frac{9}{10} \div \frac{3}{5}$

2. Discuss with your partner:

   a. Where in the diagram for $\frac{3}{4} \div \frac{1}{8}$ can we see the multiplication by the denominator 8?

   b. Where in the diagram for $\frac{9}{10} \div \frac{3}{5}$ can we see the division by the numerator 3?

   c. Where in each diagram do you see the quotient?

3. Complete this sentence based on what you noticed:

   To divide a number n by a fraction $\frac{a}{b}$, we can multiply n by _____ and then divide the product by _____.

4. Select **all** the equations that represent the sentence you completed.

   - $n \div \frac{a}{b} = n \cdot b \div a$
   - $n \div \frac{a}{b} = n \cdot a \div b$
   - $n \div \frac{a}{b} = n \cdot \frac{a}{b}$
   - $n \div \frac{a}{b} = n \cdot \frac{b}{a}$

11.3 Dividing with or without an Algorithm

Calculate at least four quotients. Show your reasoning.

1. $\frac{8}{9} \div 4$

2. $\frac{9}{12} \div \frac{6}{12}$

3. $3\frac{1}{3} \div \frac{2}{9}$

4. $\frac{9}{2} \div \frac{3}{8}$

5. $1\frac{2}{5} \div 3$

6. $6\frac{1}{4} \div \frac{10}{3}$

Are you ready for more?

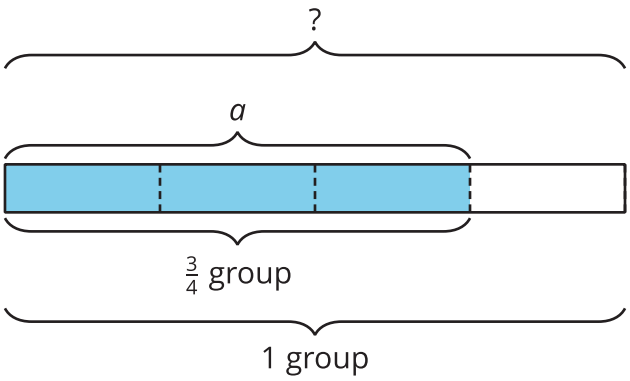
Suppose you have a quart of grape juice and a quart of milk. You pour 1 cup of the grape juice into the milk and mix it up. Then you pour 1 cup of this mixture back into the grape juice.

Which liquid is more contaminated? Explain how you know. (Note: 1 quart is equal to 4 cups.)

Lesson 11 Summary

We can think of the division $a \div \frac{3}{4} = ?$ in terms of finding the size of 1 group: "If there is a in $\frac{3}{4}$ of a group, how much is in 1 group?" or "If $\frac{3}{4}$ of a number is a, what is that number?"

On a tape diagram, we can show $\frac{3}{4}$ of a group having a value of a and the whole group having an unknown value.



If $\frac{3}{4}$ of a number is a, then to find the number, we can first divide a by 3 to find $\frac{1}{4}$ of the number. Then we multiply the result by 4 to find the number.

These steps can be written as: $a \div 3 \cdot 4$. Dividing by 3 is the same as multiplying by $\frac{1}{3}$, so we can also write the steps as: $a \cdot \frac{1}{3} \cdot 4$, which is $a \cdot \frac{4}{3}$.

In other words:

$$a \div \frac{3}{4} = a \cdot \frac{4}{3}$$

In general, dividing a number by a fraction $\frac{c}{d}$ is the same as multiplying the number by $\frac{d}{c}$, which is the **reciprocal** of the fraction. Reciprocals are numbers that when multiplied equal 1.