Lines in Triangles

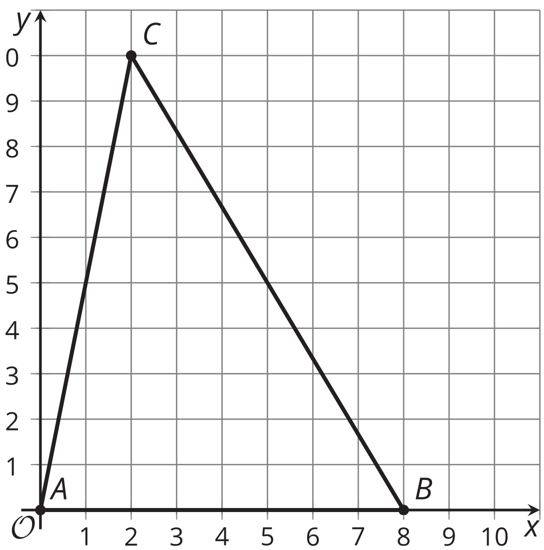
Let's investigate more special segments in triangles.

17.1 Folding Altitudes

Draw a triangle on tracing paper. Fold the altitude from each vertex.

17.2 Altitude Attributes

Triangle ABC is graphed.



1. Find the slope of each side of the triangle.

2. Find the slope of each altitude of the triangle.

3. Sketch the altitudes. Label the point of intersection, H.
4. Write equations for all 3 altitudes.

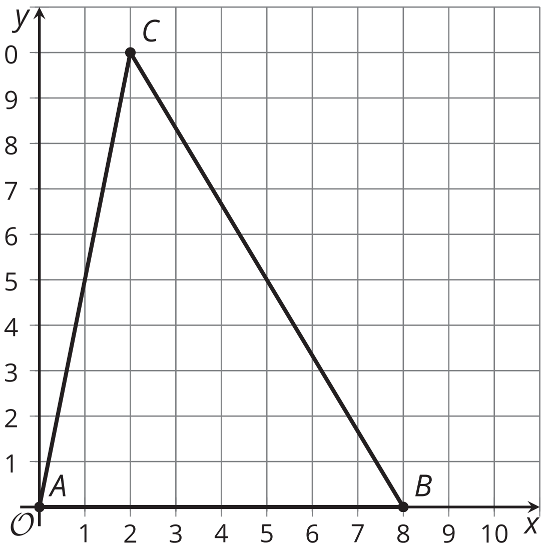
5. Use the equations to find the coordinates of H, and verify algebraically that the altitudes all intersect at H.

Are you ready for more?

Any triangle ABC can be translated, rotated, and dilated so that the image A' lies on the origin, B' lies on the point $(1, 0)$, and C' has position (a, b). Use this as a starting point to prove that the altitudes of any triangle will all meet at the same point.

17.3 Percolating on Perpendicular Bisectors

Triangle ABC is graphed.



1. Find the midpoint of each side of the triangle.

2. Sketch the perpendicular bisectors, using an index card to help draw 90-degree angles. Label the intersection point as P.
3. Write equations for all 3 perpendicular bisectors.

4. Use the equations to find the coordinates of P, and verify algebraically that the perpendicular bisectors all intersect at P.

17.4 Perks of Perpendicular Bisectors

Consider triangle ABC from an earlier activity.

1. What is the distance from A to P, the intersection point of the perpendicular bisectors of the triangle's sides? Round to the nearest tenth.

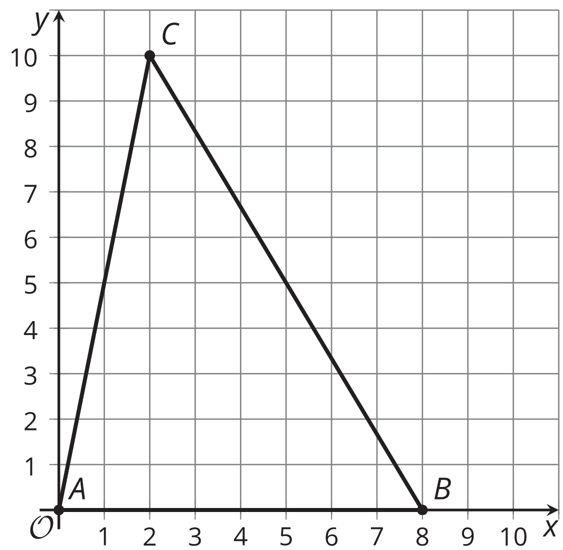
2. Write the equation of a circle with center P and radius AP.

3. Construct the circle. What do you notice?

4. Verify your hypothesis algebraically.

17.5 Amazing Points

Consider triangle ABC from earlier activities.



1. Plot point H, the intersection point of the altitudes.
2. Plot point P, the intersection point of the perpendicular bisectors.
3. Find the point where the 3 medians of the triangle intersect. Plot this point and label it J.

4. What seems to be true about points H, P, and J? Prove that your observation is true.

17.6 Tiling the (Coordinate) Plane

A tessellation covers the entire plane with shapes that do not overlap or leave gaps.

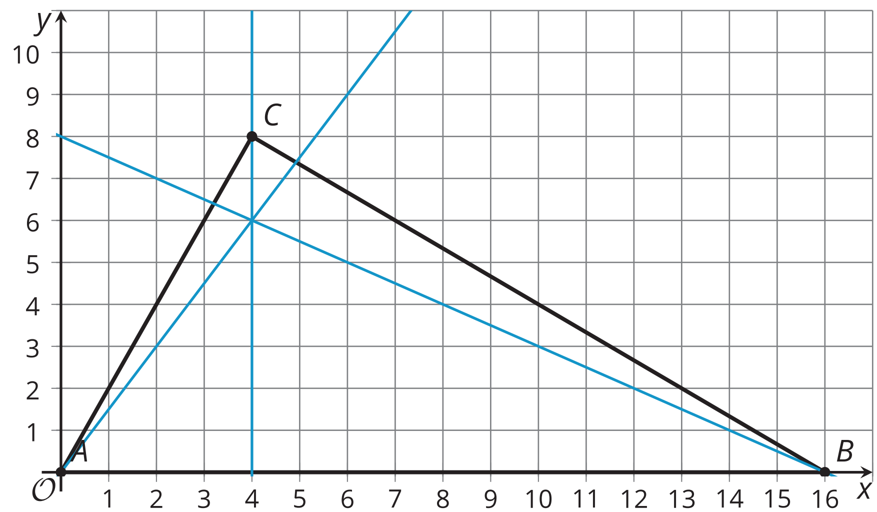
1. Tile the plane with congruent rectangles:
   a. Draw the rectangles on your grid.
   b. Write the equations for lines that outline 1 rectangle.

2. Tile the plane with congruent right triangles:
   a. Draw the right triangles on your grid.
   b. Write the equations for lines that outline 1 right triangle.

3. Tile the plane with any other shapes:
   a. Draw the shapes on your grid.
   b. Write the equations for lines that outline 1 of the shapes.

Lesson 17 Summary

The 3 medians of a triangle always intersect in 1 point. We can use coordinate geometry to show that the altitudes of a triangle intersect in 1 point, too. The 3 altitudes of triangle ABC are shown here. They appear to intersect at the point $(4, 6)$. By finding their equations, we can prove this is true.



The slopes of sides AB, BC, and AC are 0, $-\frac{2}{3}$, and 2. The altitude from C is the vertical line $x = 4$. The slope of the altitude from A is $\frac{3}{2}$. Since the altitude goes through $(0, 0)$, its equation is $y = \frac{3}{2}x$. The slope of the altitude from B is $-\frac{1}{2}$. Following this slope over to the y-axis we can see that the y-intercept is 8. So the equation for this altitude is $y = -\frac{1}{2}x + 8$.

We can now verify that $(4, 6)$ lies on all 3 altitudes by showing that the point satisfies the 3 equations. By substitution we see that each equation is true when $x = 4$ and $y = 6$.

