Unit 4, Lesson 12

The Number e

Let's learn about the number e.

12.1 Matching Situations and Equations

Match each equation to a situation it represents. Be prepared to explain how you know. Not all equations have a match.

$f(t) = 400 \cdot (0.5)^{0.1t}$

$g(t) = 400 \cdot (1.25)^{0.1t}$

$h(t) = 400 \cdot (0.75)^{0.1t}$

$j(t) = 400 \cdot (2)^{10t}$

$k(t) = 400 \cdot (2)^{0.1t}$

1. A scientist begins an experiment with 400 bacteria in a petri dish. The population doubles every 10 hours. The function gives the number of bacteria t hours since the experiment began.

2. An erosion model begins with 400 kilograms of sand. The amount of sand decreases by 25% every 10 days after the experiment begins. The function gives the amount of sand left t days after the experiment begins.

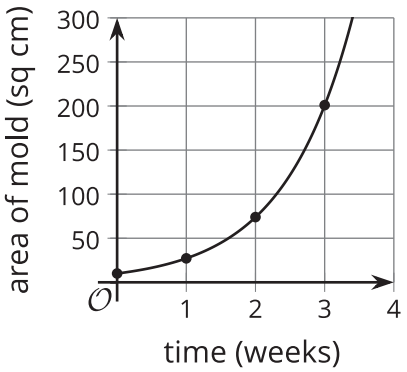
3. The half-life of a radioactive element is 10 years. There are 400 g of the element in a sample when it is first studied. The function gives the amount of the element remaining t years later.

4. In a lake, the population of a species of fish is 400. The population is expected to grow by 25% in the next decade. The function gives the number of fish in the lake t years after it was 400.

12.2 Notice and Wonder: Moldy Growth

A spot of mold is found on a basement wall. Its area is about 10 square centimeters. Here are three representations of a function that models how the mold is growing.

$a(t) = 10 \cdot e^t$



time (weeks)	area of mold (sq cm)
0	10
1	27
2	74
3	201
4	546

What do you notice? What do you wonder?

12.3 $(1 + \text{tiny})^{\text{huge}}$

1. Here are some functions. For each function, describe, in words, the outputs for very tiny, positive values of x and for very large values of x.

   $a(x) = 1^x$

   $b(x) = -x$

   $d(x) = \frac{1}{x}$

   $f(x) = \left(\frac{1}{x}\right)^x$

   $g(x) = \left(1 + \frac{1}{x}\right)^x$

   $h(x) = e^x$

   $k(x) = 1 + x$

2. Remember that $e \approx 2.718$. What does the function g have to do with the number e?

3. What do you notice about the relationship between h and k for very small, positive values of x?

Are you ready for more?

Complete the table to show the value of each expression to the nearest hundred-thousandth. Two entries have already been completed as an example.

x	2^x	e^x	3^x	$1 + x$
0.1	1.07177	1.10517		
0.01				
0.001				
0.0001				

What do you notice about the values in the table? Which columns are the closest in value?

Lesson 12 Summary

Scientists, economists, engineers, and others often use the number e in their mathematical models. What is e?

e is an important constant in mathematics, just like the constant π, which is important in geometry. The value of e is approximately 2.718. Like π, the number e is irrational, so it can't be represented as a fraction, and its decimal representation never repeats or terminates. The number is named after the 18th-century mathematician Leonhard Euler and is sometimes called *Euler's number*.

e has many useful properties and it arises in situations involving exponential growth or decay, so e often appears in exponential functions.