Lesson 12: Arithmetic with Complex Numbers

- Let's work with complex numbers.

12.1: Math Talk: Telescoping Sums

Find the value of these expressions mentally.

$2 - 2 + 20 - 20 + 200 - 200$

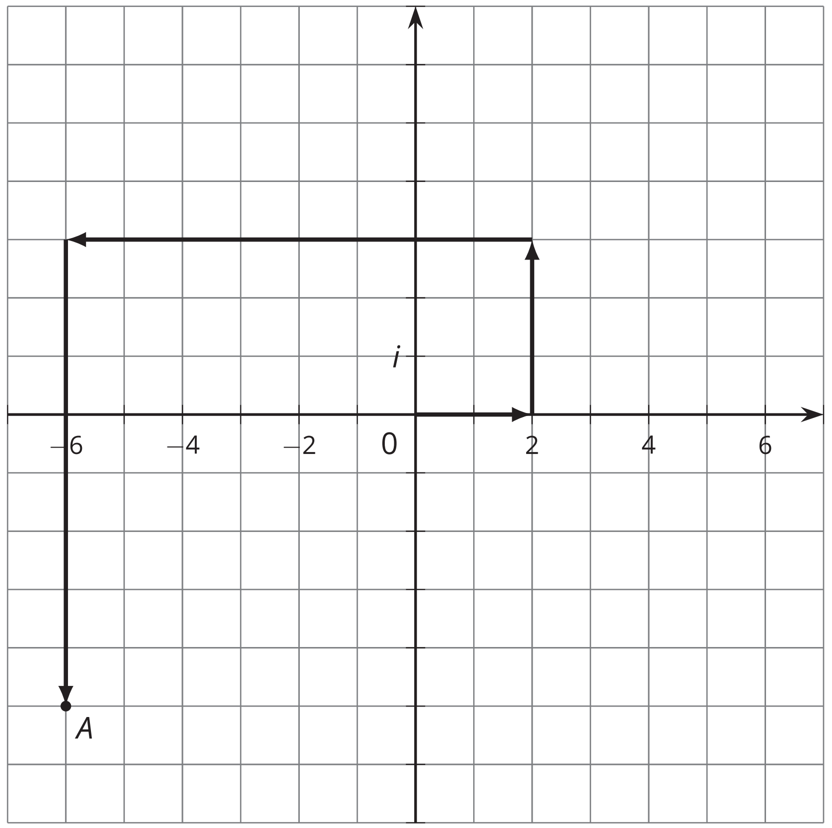
$100 - 50 + 10 - 10 + 50 - 100$

$3 + 2 + 1 + 0 - 1 - 2 - 3$

$1 + 2 + 4 + 8 + 16 + 32 - 16 - 8 - 4 - 2 - 1$

12.2: Adding Complex Numbers

1. This diagram represents $(2 + 3i) + (\text{-}8 - 8i)$.



a. How do you see $2 + 3i$ represented?

b. How do you see $\text{-}8 - 8i$ represented?

c. What complex number does A represent?

d. Add “like terms” in the expression $(2 + 3i) + (\text{-}8 - 8i)$. What do you get?

2. Write these sums and differences in the form $a + bi$, where a and b are real numbers.
   a. $(-3 + 2i) + (4 - 5i)$ (Check your work by drawing a diagram.)

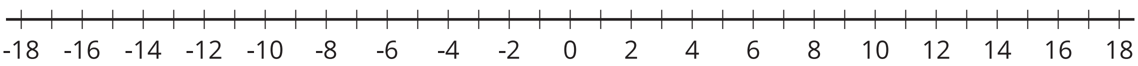
   b. $(-37 - 45i) + (11 + 81i)$

   c. $(-3 + 2i) - (4 - 5i)$

   d. $(-37 - 45i) - (11 + 81i)$

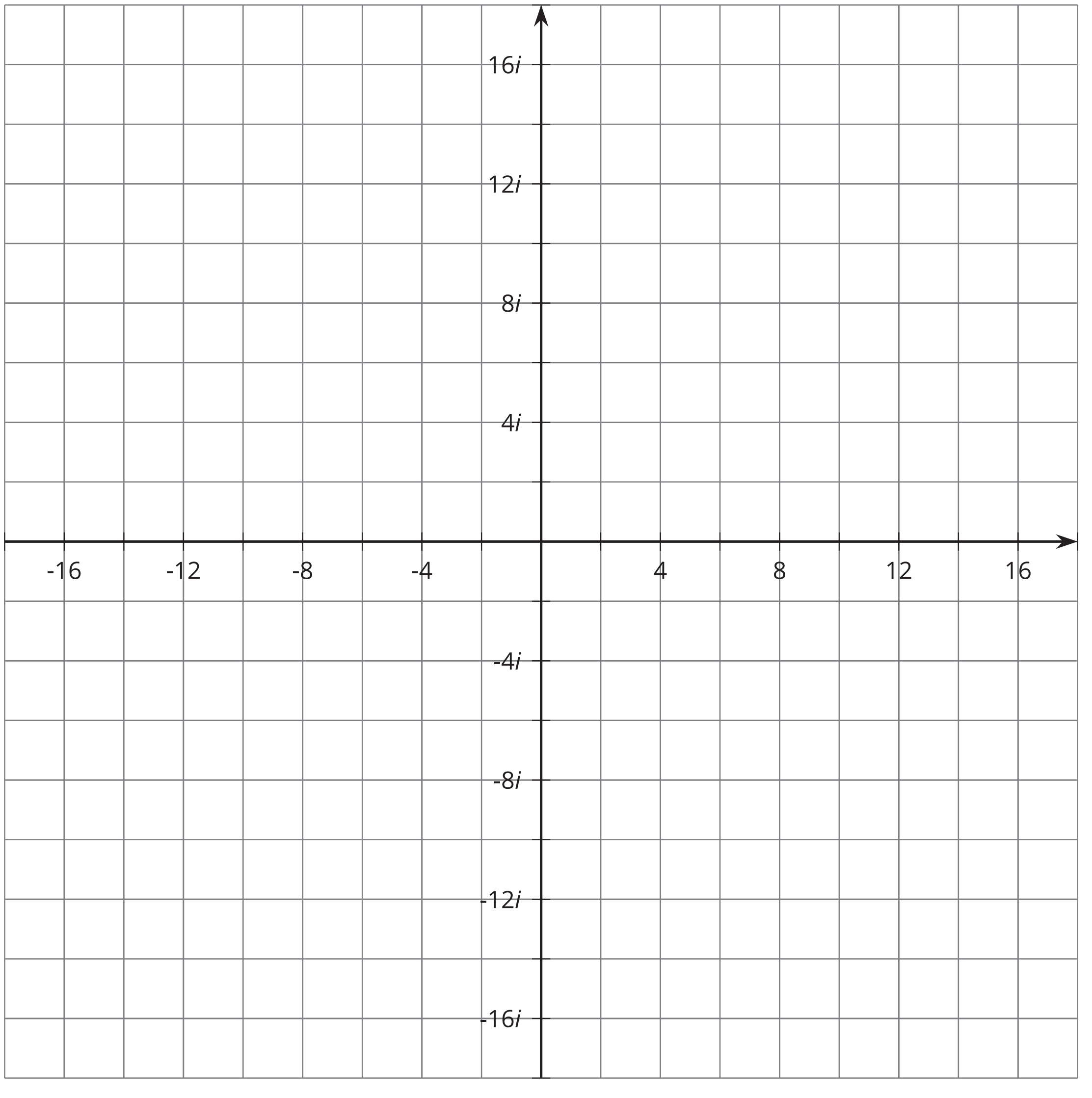
12.3: Multiplication on the Complex Plane

1. Draw points to represent 2, 2^2, 2^3, and 2^4 on the real number line.



2. a. Write $2i$, $(2i)^2$, $(2i)^3$, and $(2i)^4$ in the form $a + bi$.

b. Plot $2i$, $(2i)^2$, $(2i)^3$, and $(2i)^4$ on the complex plane.



Are you ready for more?

1. If a and b are positive numbers, is it true that $\sqrt{ab} = \sqrt{a}\sqrt{b}$? Explain how you know.

2. If a and b are negative numbers, is it true that $\sqrt{ab} = \sqrt{a}\sqrt{b}$? Explain how you know.

Lesson 12 Summary

When we add a real number with an imaginary number, we get a complex number. We usually write complex numbers as:

$$a + bi$$

where a and b are real numbers. We say that a is the real part and bi is the imaginary part.

To add (or subtract) two complex numbers, we add (or subtract) the real parts and add (or subtract) the imaginary parts. For example:

$$(2 + 3i) + (4 + 5i) = (2 + 4) + (3i + 5i) = 6 + 8i$$

$$(2 + 3i) - (4 + 5i) = (2 - 4) + (3i - 5i) = \text{-}2 - 2i$$

In general:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

and:

$$(a + bi) - (c + di) = (a - c) + (b - d)i$$

When we raise an imaginary number to a power, we can use the fact that $i^2 = \text{-}1$ to write the result in the form $a + bi$. For example, $(4i)^3 = 4i \cdot 4i \cdot 4i$. We can group the i factors together to see how to rewrite this.

$$\begin{aligned} 4i \cdot 4i \cdot 4i &= (4 \cdot 4 \cdot 4) \cdot (i \cdot i \cdot i) \\ &= 64 \cdot (i^2 \cdot i) \\ &= 64 \cdot \text{-}1 \cdot i \\ &= \text{-}64i \end{aligned}$$

So in this example, a is 0 and b is -64.