How Many Solutions?

Let's solve equations with different numbers of solutions.

8.1 Matching Solutions

Match each equation with the number of values that solve the equation.

1. $12(x-3)+18=6(2x-3)$
2. $12(x-3)+18=4(3x-3)$
3. $12(x-3)+18=4(2x-3)$

- true for only 1 value
- true for no values
- true for any value

8.2 Card Sort: Solutions

Your teacher will give you a set of cards containing equations.

- Sort the cards into categories of your choosing.
- Describe the defining characteristics of the categories, and be prepared to share your reasoning with the class.

8.3 Make Use of Structure

For each equation, determine whether it has no solutions, exactly one solution, or is true for all values of x (and has infinitely many solutions). If an equation has one solution, solve the equation to find the value of x that makes the statement true.

1. a. $6x + 8 = 7x + 13$

   b. $6x + 8 = 2(3x + 4)$

   c. $6x + 8 = 6x + 13$

2. a. $\frac{1}{4}(12 - 4x) = 3 - x$

   b. $x - 3 = 3 - x$

   c. $x - 3 = 3 + x$

3. a. $-5x - 3x + 2 = -8x + 2$

   b. $-5x - 3x - 4 = -8x + 2$

   c. $-5x - 4x - 2 = -8x + 2$

4. a. $4(2x - 2) + 2 = 4(x - 2)$

   b. $4x + 2(2x - 3) = 8(x - 1)$

   c. $4x + 2(2x - 3) = 4(2x - 2) + 2$

5. a. $x - 3(2 - 3x) = 2(5x + 3)$

   b. $x - 3(2 + 3x) = 2(5x - 3)$

   c. $x - 3(2 - 3x) = 2(5x - 3)$

6. What do you notice about equations with one solution? How is this different from equations with no solutions and equations that are true for every x?

Are you ready for more?

Consecutive numbers follow one right after the other. An example of three consecutive numbers is 17, 18, and 19. Another example is -100, -99, -98.

1. Choose any set of three consecutive numbers. Find their average. What do you notice?

2. Find the average of another set of three consecutive numbers. What do you notice?

3. Explain why the thing you noticed must always work, or find a counterexample.

Lesson 8 Summary

Sometimes it's possible to look at the structure of an equation and tell if it has infinitely many solutions or no solutions. For example, look at

$$2(12x + 18) + 6 = 18x + 6(x + 7).$$

Using the distributive property on the left and right sides, we get

$$24x + 36 + 6 = 18x + 6x + 42.$$

From here, collecting like terms gives us

$$24x + 42 = 24x + 42.$$

Without doing any more moves, we know that this equation is true for any value of x because the left and right sides of the equation are the same.

Similarly, we can sometimes use structure to tell if an equation has no solutions. For example, look at

$$6(6x + 5) = 12(3x + 2) + 12.$$

If we think about each move as we go, we can stop when we realize there is no solution:

$$\begin{aligned} \frac{1}{6} \cdot 6(6x + 5) &= \frac{1}{6} \cdot (12(3x + 2) + 12) && \text{Multiply each side by } \frac{1}{6}. \\ 6x + 5 &= 2(3x + 2) + 2 && \text{Distribute } \frac{1}{6} \text{ on the right side.} \\ 6x + 5 &= 6x + 4 + 2 && \text{Distribute 2 on the right side.} \end{aligned}$$

Because the coefficient of x is 6 on each side, we know that there is either no solution or infinitely many solutions. The last move makes it clear that the **constant terms** on each side, 5 and $4 + 2$, are not the same. Because adding 5 to an amount is always less than adding $4 + 2$ to that same amount, we know that there are no solutions.

Doing moves to keep an equation balanced is a powerful part of solving equations, but thinking about what the structure of an equation tells us about the solutions is just as important.

