



# Equations and Their Solutions

Let's recall what we know about solutions to equations.

## 4.1 What Is a Solution?

Cement is made by mixing water and concrete mix. The amount of water,  $w$ , added to the concrete mix can affect the strength of the material. One liter of water weighs 2.2 pounds.

The equation  $2.2w + 80 = 86.05$  represents the relationship between these quantities.

1. What could the 80 represent in this situation?
  
  
  
  
  
  
  
  
  
  
2. Priya said that neither 2 nor 3 could be the solution to the equation. Explain why she is correct.
  
  
  
  
  
  
  
  
  
  
3. Find the solution to the equation.

## 4.2

# Weekend Earnings

Jada has time on the weekends to earn some money. A local bookstore is looking for someone to help sort books and will pay \$12.20 an hour. To get to and from the bookstore on a work day, however, Jada would have to spend \$7.15 on bus fare.

1. Write an equation that represents Jada's take-home earnings in dollars,  $E$ , if she works at the bookstore for  $h$  hours.
2. One day, Jada takes home \$90.45 after working  $h$  hours and after paying the bus fare. Write an equation to represent this situation.
3. Is 4 a solution to the last equation you wrote? What about 7?
  - If so, be prepared to explain how you know one or both of them are solutions.
  - If not, be prepared to explain why they are not solutions. Then, find the solution.
4. In this situation, what does the solution to the equation tell us?



### Are you ready for more?

Jada has a second option to earn money—she could help some neighbors with errands and computer work for \$11 an hour. After reconsidering her schedule, Jada realizes that she has about 9 hours available to work one day of the weekend.

Which option should she choose—sorting books at the bookstore or helping her neighbors? Explain your reasoning.



## 4.3

# Carbon Dioxide Production

One gallon of gasoline produces about 20 pounds of carbon dioxide. One gallon of pure ethanol produces about 13 pounds of carbon. A car engine that can run on gasoline or ethanol produces 100 pounds of carbon dioxide from  $g$  gallons of gasoline and  $c$  gallons of ethanol.

The equation  $20g + 13c = 100$  represents the relationship between these quantities.

- Determine if each pair of values could be the number of gallons of gasoline and ethanol burned in the car engine. Be prepared to explain your reasoning.
  - 5 gallons of gasoline and 7.7 gallons of ethanol
  
  - 3.05 gallons of gasoline and 3 gallons of ethanol
  
  - 2 gallons of gasoline and 4.5 gallons of ethanol
- If the car engine burned 4 gallons of gasoline, how many gallons of ethanol were burned? Show your reasoning.
  
  
  
  
  
  
  
  
  
  
- In this situation, what does a solution to the equation  $20g + 13c = 100$  tell us? Give an example of a solution.



## Lesson 4 Summary

An equation that contains only one unknown quantity or one quantity that can vary is called an *equation in one variable*.

For example, the equation  $2\ell + 2w = 72$  represents the relationship between the length,  $\ell$ , and the width,  $w$ , of a rectangle that has a perimeter of 72 units. If we know that the length is 15 units, we can rewrite the equation as:

$$2(15) + 2w = 72.$$

This is an equation in one variable, because  $w$  is the only quantity that we don't know. To solve this equation means to find a value of  $w$  that makes the equation true.

In this case, 21 is the solution because substituting 21 for  $w$  in the equation results in a true statement.

$$\begin{aligned}2(15) + 2w &= 72 \\2(15) + 2(21) &= 72 \\30 + 42 &= 72 \\72 &= 72\end{aligned}$$

An equation that contains two unknown quantities or two quantities that vary is called an *equation in two variables*. A solution to such an equation is a *pair* of numbers that makes the equation true.

Suppose Tyler spends \$45 on T-shirts and socks. A T-shirt costs \$10 and a pair of socks costs \$2.50. If  $t$  represents the number of T-shirts and  $p$  represents the number of pairs of socks that Tyler buys, we can represent this situation with the equation:

$$10t + 2.50p = 45$$

This is an equation in two variables. More than one pair of values for  $t$  and  $p$  make the equation true.

$$t = 3 \text{ and } p = 6$$

$$\begin{aligned}10(3) + 2.50(6) &= 45 \\30 + 15 &= 45 \\45 &= 45\end{aligned}$$

$$t = 4 \text{ and } p = 2$$

$$\begin{aligned}10(4) + 2.50(2) &= 45 \\40 + 5 &= 45 \\45 &= 45\end{aligned}$$

$$t = 2 \text{ and } p = 10$$

$$\begin{aligned}10(2) + 2.50(10) &= 45 \\20 + 25 &= 45 \\45 &= 45\end{aligned}$$

In this situation, one constraint is that the combined cost of shirts and socks must equal \$45. Solutions to the equation are pairs of  $t$  and  $p$  values that satisfy this constraint.

Combinations such as  $t = 1$  and  $p = 10$  or  $t = 2$  and  $p = 7$  are *not* solutions because they don't meet the constraint. When these pairs of values are substituted into the equation, they result in statements that are false.

