Unit 8, Lesson 16

Solving Inequalities with Absolute Values

Let’s solve absolute value inequalities.

16.1 Interpreting Absolute Value Inequalities

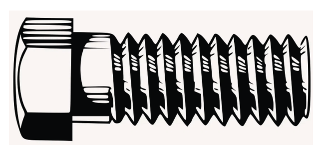
Describe what each of these expressions mean.

1. $|x-8|=4$

2. $|x-8|<4$

3. $|x-8|\geq 4$

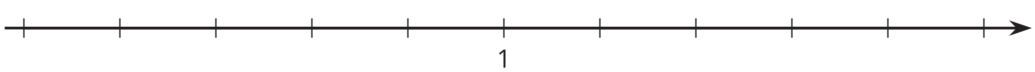
16.2 A Threaded Bolt



The part that winds around on a bolt is called the “thread.” In order to work correctly, the threads of a certain type of bolt must be about 1 millimeter apart from one another. Due to the way they are made, it is difficult to get the threads exactly 1 millimeter apart, but they will not work correctly if they are too far off. This type of bolt is acceptable if the threads are within 0.1 millimeter of what it is supposed to be.

1. What are 3 different distances between threads that would be acceptable for this type of bolt?

2. Use the number line to draw all of the distances between threads that are allowed.



3. Let x represent the distance between threads. Complete the two inequalities that describe what must be true for all bolts of this type that are acceptable.

   $x \leq$ ________ and $x \geq$ ________

4. Use absolute value to write these inequalities as a single inequality that expresses the distance from 1 millimeter that is acceptable.

5. Write an inequality using absolute value that represents threads for bolts that are unacceptable.

16.3 Solving Absolute Value Inequalities

Graph the solution to each inequality on a number line.

1. $|x - 5| \leq 1$

2. $3 > |x + 3|$

3. $|x - 0.5| \geq 2$

4. $|x - 10| > \frac{2}{3}$

Are you ready for more?

Use the piecewise definition of absolute value to write two new inequalities to solve each one of the inequalities here, then solve the new inequalities you have created.

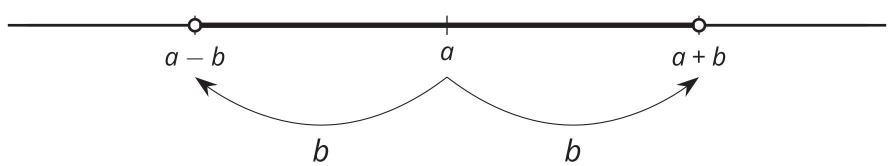
1. $|2x - 3| < 5$

2. $2|x - 1| \geq 10$

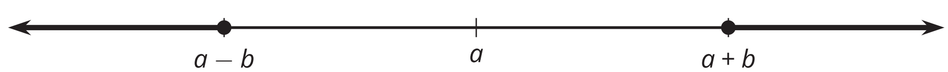
Lesson 16 Summary

In many real-world situations, it is okay if an item is not exactly the perfect length or weight. There is a range of values that will work: The length or weight of the item can be within a certain amount or at least a certain amount away. In these cases, it can be useful to write the range of values that work as inequalities that include absolute values.

To be within a certain distance of a value, we can write an inequality of the form $|x - a| < b$. The solutions to this inequality can be drawn on a number line like this:



To be at least a certain distance away from a value, we can write an inequality of the form $|x - a| \geq b$. The solutions to this inequality can be drawn on a number line like this:



For example, to fence in a circular area with a radius of 100 meters, we should use the equation $C = 2\pi r$ and buy 200π meters of fencing. This is difficult to do because π is an irrational number. Maybe it's okay if we don't make a perfect circle or if it's a little off, so if we buy x amount of fence, where x is a solution to $|x - 200\pi| < 1$, we will have the right amount of fencing within 1 meter of the exact value, and that should be close enough.