Finding Distances in the Coordinate Plane

Goals

- Calculate the distance between two points in the coordinate plane by using the Pythagorean Theorem and explain (orally) the solution method.
- Generalize (orally) a method for calculating the length of a line segment in the coordinate plane using the Pythagorean Theorem.

Learning Targets

- I can find the distance between two points in the coordinate plane.
- I can find the length of a diagonal line segment in the coordinate plane.

Lesson Narrative

In this lesson, students continue to apply the Pythagorean Theorem by finding distances between points in the coordinate plane.

Students begin by finding and ordering distances between pairs of points that lie on the same vertical or horizontal line. Next, they find the distances between three points in the coordinate plane that, if connected, would make a right triangle. Students see that to find the distance between two points (or the length of the segment connecting those two points), they can draw a right triangle and use the Pythagorean Theorem (MP7).

Next, groups of students find the distance between two points, given only their coordinates. Whether students plot these points first or go straight to using an algebraic set of steps, they must think about how to generalize and communicate their method (MP6).

This lesson contains an optional activity where students find the perimeter of triangles drawn on a coordinate plane without a grid. This activity can be useful if students need additional practice finding the length of a segment that does not lie along a gridline.

Standards

Addressing	**8.G.B.8**
Building Toward	**8.G.B.8**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Student Facing Learning Goals

Let's find distances in the coordinate plane.

11.1 Closest Distance

Warm-up

5 min

Activity Narrative

The purpose of this *Warm-up* is for students to find the distance between two points on the same horizontal or vertical line in the coordinate plane. Students are intentionally given only the coordinates and no graph. This encourages them to reason about the relative locations of the points, which will help them determine the distance between two points in the coordinate plane using the Pythagorean Theorem in a following activity.

Standards

Building Toward **8.G.B.8**

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by 1 minute to compare their responses with a partner. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Activate or supply background knowledge. Provide students with access to a blank coordinate plane.
Supports accessibility for: Visual-Spatial Processing, Organization

Student Task Statement

1. Order the following pairs of coordinates from closest to farthest apart. Be prepared to explain your reasoning.
   a. $(2, 4)$ and $(2, 10)$
   b. $(-3, 6)$ and $(5, 6)$
   c. $(-12, -12)$ and $(-12, -1)$
   d. $(7, 0)$ and $(7, -9)$
   e. $(1, -10)$ and $(-4, -10)$
2. Name another pair of coordinates that would be closer together than the first pair on your list.
3. Name another pair of coordinates that would be farther apart than the last pair on your list.

Student Response

1. e. (5 units), a. (6 units), b. (8 units), d. (9 units), c. (11 units)
2. Answers vary. Sample response: $(2, 4)$ and $(2, 8)$
3. Answers vary. Sample response: $(12, -10)$ and $(-4, -10)$

Activity Synthesis

Invite students to share how they ordered the pairs of coordinates from closest to furthest apart. Record and display the responses for all to see. After the class agrees on the correct order, ask students to share the distance between a few of the pairs of coordinates and their strategy for finding that distance. Ask 2–3 students to share pairs of coordinates they found that would have a closer or further distance than the ones in the list.

If not brought up in students' explanations, consider asking the following questions:

- "What does it mean that each pair has one coordinate that is the same?" (The two points are either on the same horizontal or vertical line.)
- "How did you decide on which coordinate to subtract?" (I subtracted the coordinates that were different.)
- "Why didn't you need to subtract the other?" (The difference would be 0.)
- "Could we represent this distance with a line segment? (Yes, we could draw in a line segment that connects the two points.)
- "Would your strategy work for any pair of coordinates?" (No, it would only work for a pair of coordinates that both lie on the same horizontal or vertical line.)
- "Which pairs would it work for? Which pairs are we not sure if it would work for?" (It would not work for pairs of coordinates that are not on the same horizontal or vertical line.)

11.2 How Far Apart?

10 min

Activity Narrative

In this activity, students find distances between points in the coordinate plane. The points are shown on a coordinate plane without a grid to encourage reasoning about the coordinates rather than counting grid squares. If connected, the three points are the vertices of a right triangle, which will help students recognize this situation as one where the Pythagorean Theorem can be used.

Standards

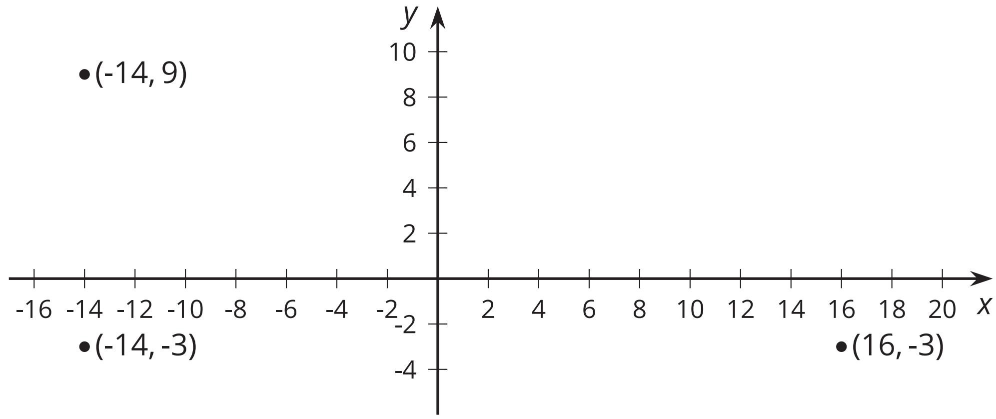
Addressing **8.G.B.8**

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time followed by partner discussion. Then follow with a whole-class discussion.

Student Task Statement

Find the distances between the three points shown.



Student Response

12, 30, $\sqrt{1044}$

Activity Synthesis

The goal of this discussion is for students to see how they can find the distance between any two points in the coordinate plane by adding a third coordinate that allows them to draw a right triangle.

Begin by displaying the image from the *Task Statement*. Before students share their solutions, display an incorrect solution based on a common error you observe for finding the distance between points with negative coordinates. For example, "The distance between $(-14, 9)$ and $(-14, -3)$ is 6, because $9 - 3 = 6$." Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who reference the points in the coordinate plane to show that the distances are incorrect. Then invite groups to share their solutions.

Next, draw two points on the coordinate plane, for example $(-3, -4)$ and $(2, 7)$. Ask students how we can use the problem we just solved to find the distance between these two points. (Draw the point $(2, -4)$ or $(-3, 7)$ and draw a right triangle between the points, then use the Pythagorean Theorem.)

If time allows, ask students to calculate the distance between these two new points.

11.3 Perimeters with Pythagoras

Optional

15 min

Activity Narrative

In this optional activity, students calculate the perimeters of two triangles on the coordinate plane using the Pythagorean Theorem. Use this activity if time allows and students need practice calculating the distance between two points in the coordinate plane.

Standards

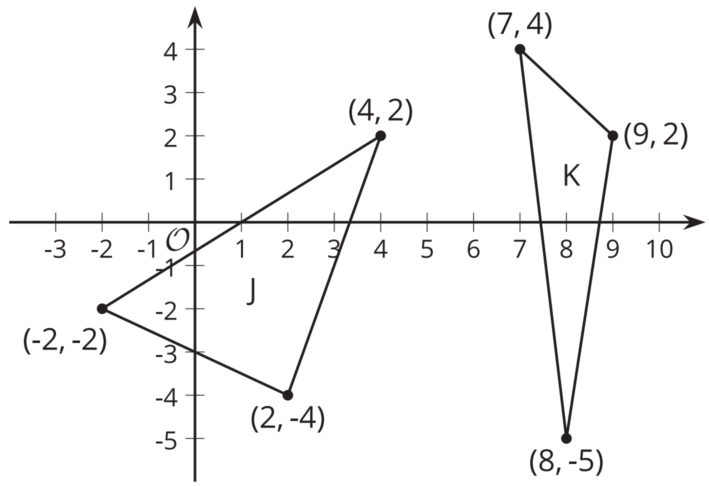
Addressing **8.G.B.8**

Launch

Arrange students in groups of 2. Display the image from the *Task Statement* for all to see and then ask the class which figure they think has the longer perimeter. Display the results for all to see. Tell the class that each partner will now calculate the perimeter of one of the figures.

Student Task Statement

1. Which triangle do you think has the longer perimeter? Be prepared to explain your reasoning.
2. Select one triangle and calculate its perimeter. Your partner will calculate the perimeter of the other. Were you correct about which figure had the longer perimeter?



Student Response

1. Sample reasoning: I think Triangle J has a longer perimeter because it looks like it has a larger area.
2. Triangle J has a perimeter of approximately 18 units. The perimeter is the sum $\sqrt{2^2+6^2}+\sqrt{2^2+4^2}+\sqrt{4^2+6^2}=\sqrt{40}+\sqrt{20}+\sqrt{52}\approx 18$. Triangle K has a perimeter of approximately 19 units. The perimeter is the sum $\sqrt{9^2+1^2}+\sqrt{7^2+1^2}+\sqrt{2^2+2^2}=\sqrt{82}+\sqrt{50}+\sqrt{8}\approx 19$.

Are You Ready for More?

Quadrilateral $ABCD$ has vertices at $A=(-5,1)$, $B=(-4,4)$, $C=(2,2)$, and $D=(1,-1)$.

1. Use the Pythagorean Theorem to find the lengths of sides AB, BC, CD, and AD.
2. Use the Pythagorean Theorem to find the lengths of the two diagonals, AC and BD.
3. Explain why quadrilateral $ABCD$ is a rectangle.

Extension Student Response

1. Use the Pythagorean Theorem to find the length of each segment. Segment AB has length $\sqrt{10}$ because $(AB)^2=1^2+3^2$. Segment CD also has length $\sqrt{10}$ because the right triangle used to find length AB is congruent to the right triangle used to find length CD. The length of segment AD is $\sqrt{40}$ because $(AD)^2=6^2+2^2$. The triangle used to calculate the length of segment BC is congruent to the one used to calculate AD, so the length of segment BC is also $\sqrt{40}$.
2. The length of segment AC is $\sqrt{50}$ because $AC^2=7^2+1^2$. The length of segment BD is also $\sqrt{50}$ because $(BD)^2=5^2+5^2$.

3. The figure $ABCD$ is a rectangle because it has four right angles. For example, the angle at A is a right angle by the converse of the Pythagorean Theorem, since we have that $(AD)^2 + (AB)^2 = (BD)^2$.

Activity Synthesis

Ask students which figure they think has the longer perimeter to see how the results have changed. For each triangle, select 1–2 students to share their calculations.

11.4 Finding the Right Distance

15 min

Activity Narrative

The purpose of this task is for students to think about a general method for finding the distance between two points in the coordinate plane. Students do not need to formalize this into a more traditional representation of the distance formula, but they should be able to communicate precisely about their methods (MP6).

In groups of 4, each student will find the distance between two coordinate pairs and then share how they completed their calculations. The coordinates are carefully chosen so that the distances are all equal. Each pair represents a possible diameter for a circle centered at $(2, -2)$ with radius 5, though students do not need to know this in order to complete their calculations.

Identify students who clearly explain their thinking as they work with their group. Monitor for any groups that discover the points are all on the perimeter of a circle.

Standards

Addressing **8.G.B.8**

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 4. Tell students that each person will calculate the distance between a different set of points and once everyone in their group has finished, they will share how they did their calculations and then answer the problems. Encourage students to listen carefully to the ideas of other members of their group in order to write a clear explanation for the second question.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Provide tools to facilitate information processing or computation, enabling students to focus on key mathematical ideas. For example, allow students to use calculators and a blank coordinate plane to support their reasoning.
Supports accessibility for: Memory, Conceptual Processing

Student Task Statement

Have each person in your group select one of the sets of coordinate pairs shown. Then calculate the distance between those two coordinates. Once the distances are calculated, have each person in the group briefly share how they did their calculations.

- $(-2, 1)$ and $(6, -5)$
- $(-1, -6)$ and $(5, 2)$
- $(-1, 2)$ and $(5, -6)$
- $(-2, -5)$ and $(6, 1)$

1. How does the distance between your coordinate pairs compare to the distances for the rest of your group?
2. In your own words, write an explanation to another student of how to find the distance between any two coordinate pairs.

Student Response

1. All distances are 10 units.
2. Sample response: For the coordinate pairs $(-2, 1)$ and $(6, -5)$, a right triangle can be drawn with the coordinate pairs as vertices. The legs of this triangle are 6 and 8. This means the distance between the coordinate pairs is given by c in the equation $6^2 + 8^2 = c^2$. Then $c = 10$ since $c^2 = 36 + 64 = 100$.

Activity Synthesis

The purpose of this discussion is for students to compare their methods for finding the distance between two points. Select 2–3 previously identified students to share how they found the distance between their points.

If any groups figured out that the points lie on a circle, ask them to share how they did so. Then ask students to find the distance between one (or both) of their points and the point $(2, -2)$ using the method they described in the second problem. If students' calculations are correct, they should get a distance of 5 units, which is the radius of the circle.

Access for English Language Learners

MLR1 Stronger and Clearer Each Time. Before the whole-class discussion, give students time to meet with 2–3 partners to share and get feedback on their first draft response to the second question "In your own words, write an explanation to another student of how to find the distance between any two coordinate pairs." Invite listeners to ask questions and give feedback that will help their partner clarify and strengthen their ideas and writing. Give students 3–5 minutes to revise their first draft based on the feedback they receive.
Advances: Writing, Speaking, Listening

Lesson Synthesis

The purpose of the discussion is to check that students understand how to use the Pythagorean Theorem to calculate distances between points in the coordinate plane. Here are some questions for discussion:

- "How can you find the distance between points in the coordinate plane?" (If they are on the same horizontal or vertical line, we just subtract the coordinates that are different. If they aren't, we can construct a right triangle and use the Pythagorean Theorem.)
- "What advice would you give someone using the Pythagorean Theorem to find the distance between two points in the coordinate plane?" (Make a sketch!)

11.5

Lengths of Line Segments

Cool-down

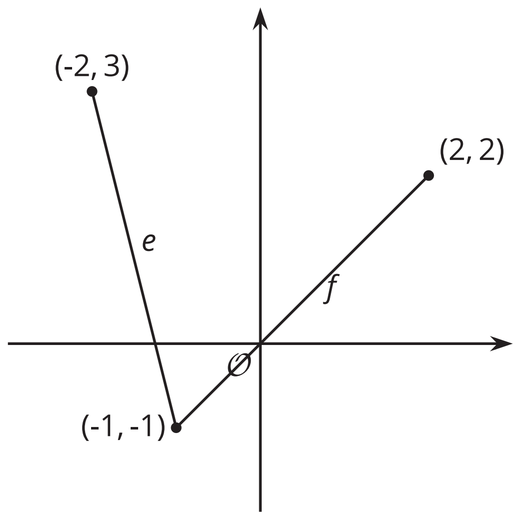
5 min

Standards

Addressing **8.G.B.8**

Student Task Statement

Calculate the exact lengths of segments e and f. Which segment is longer?



Student Response

The length of e is $\sqrt{17}$ units, and the length of f is $\sqrt{18}$ units. $e = \sqrt{1^2 + 4^2} = \sqrt{1+16} = \sqrt{17}$. $f = \sqrt{3^2 + 3^2} = \sqrt{9+9} = \sqrt{18}$. Line segment f is longer.

Responding to Student Thinking

Points to Emphasize

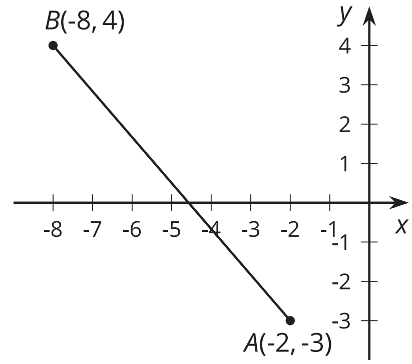
If most students struggle with finding the distance between two points, revisit how to use the Pythagorean Theorem. For example, in the practice probelm referred to here, encourage students to plot the points.

Accelerated 7, Unit 8, Lesson 13, Practice Problem 6

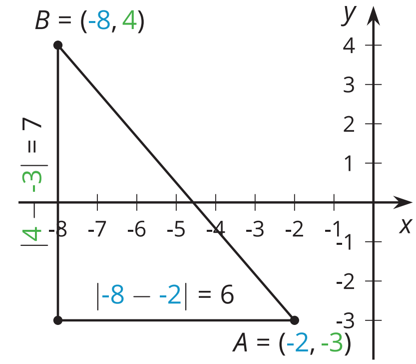
Lesson 11 Summary

We can use the Pythagorean Theorem to find the distance between any two points in the coordinate plane.

For example, if the coordinates of point A are $(-2, -3)$, and the coordinates of point B are $(-8, 4)$, the distance between them is also the length of line segment AB. It is a good idea to plot the points first.



Think of the segment AB as the hypotenuse of a right triangle. The legs can be drawn in as horizontal and vertical line segments.



The length of the horizontal leg is 6, which can be seen in the diagram. This is also the distance between the x-coordinates of A and B ($|-8 - -2| = 6$).

The length of the vertical leg is 7, which can be seen in the diagram. This is also the distance between the y-coordinates of A and B ($|4 - -3| = 7$).

Once the lengths of the legs are known, we use the Pythagorean Theorem to find the length of the hypotenuse, AB, which we can represent with c:

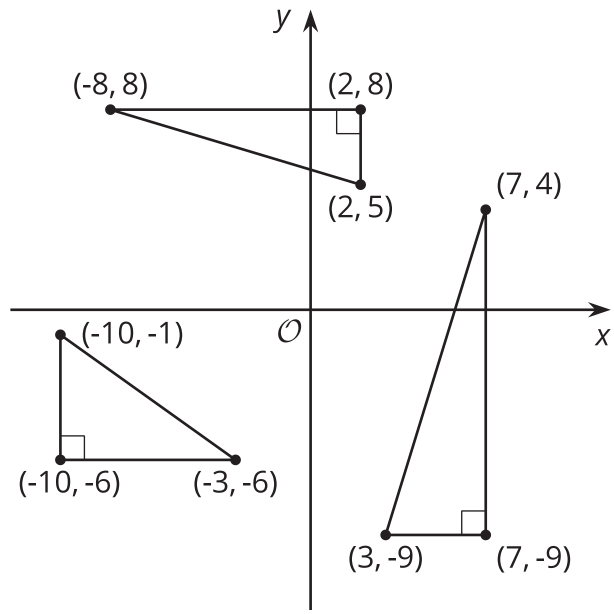
$$\begin{aligned} 6^2 + 7^2 &= c^2 \\ 36 + 49 &= c^2 \\ 85 &= c^2 \\ \sqrt{85} &= c \end{aligned}$$

This length is a little longer than 9, since 85 is a little longer than 81. Using a calculator gives a more precise answer, $\sqrt{85} \approx 9.22$.

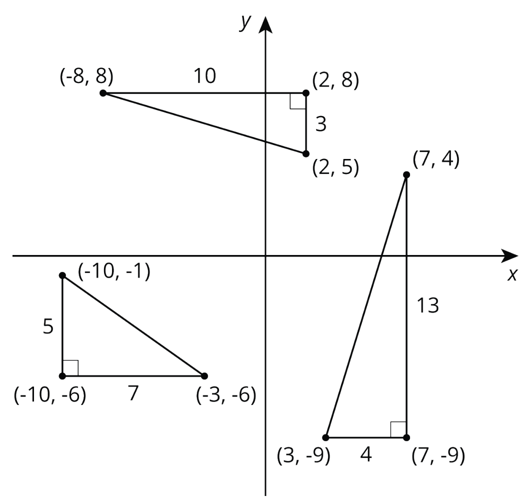
Lesson 11 Practice Problems

1 Student Task Statement

The right triangles are drawn in the coordinate plane, and the coordinates of their vertices are labeled. For each right triangle, label each leg with its length.



Solution



2 Student Task Statement

Find the distance (in units) between each pair of points. If you get stuck, try plotting the points on graph paper.

a. $M = (0, -11)$ and $P = (0, 2)$
b. $A = (0, 0)$ and $B = (-3, -4)$
c. $C = (8, 0)$ and $D = (0, -6)$

Solution

a. 13 units
b. 5 units
c. 10 units

3 from Unit 8, Lesson 8

Student Task Statement

a. Find an object that contains a right angle. This can be something in nature or something that was made by humans or machines.
b. Measure the two sides that make the right angle. Then measure the distance from the end of one side to the end of the other.
c. Draw a diagram of the object, including the measurements.
d. Use the Pythagorean Theorem to show that your object really does have a right angle.

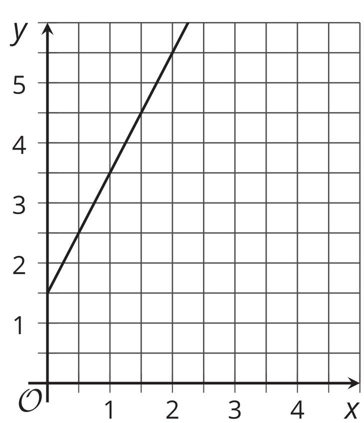
Solution

Answers vary. A correct response will include a labeled diagram and the three measurements inserted into the equation $a^2 + b^2 = c^2$ with enough work to show that the three measurements make this equation true (or close-enough, accounting for measurement error).

4 from Unit 5, Lesson 6

Student Task Statement

Write an equation for the graph.



Solution

$y = 2x + 1.5$ (or equivalent)

