



# The Quadratic Formula

Let's learn a formula for finding solutions to quadratic equations.

## 16.1 Evaluate It

Each expression represents two numbers. Evaluate the expressions and find the two numbers.

1.  $1 \pm \sqrt{49}$

2.  $\frac{8 \pm 2}{5}$

3.  $\pm\sqrt{(-5)^2 - 4 \cdot 4 \cdot 1}$

4.  $\frac{-18 \pm \sqrt{36}}{2 \cdot 3}$

## 16.2 Pesky Equations

Choose one equation to solve, either by rewriting it in factored form or by completing the square. Be prepared to explain your choice of method.

1.  $x^2 - 2x - 1.25 = 0$

2.  $5x^2 + 9x - 44 = 0$

3.  $x^2 + 1.25x = 0.375$

4.  $4x^2 - 28x + 29 = 0$



## 16.3 Meet the Quadratic Formula

Here is a formula called the **quadratic formula**.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The formula can be used to find the solutions to any quadratic equation in the form of  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are numbers and  $a$  is not 0.

This example shows how it is used to solve  $x^2 - 8x + 15 = 0$ , in which  $a = 1$ ,  $b = -8$ , and  $c = 15$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

original equation

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(15)}}{2(1)}$$

Substitute the values of  $a$ ,  $b$ , and  $c$ .

$$x = \frac{8 \pm \sqrt{64 - 60}}{2}$$

Evaluate each part of the expression.

$$x = \frac{8 \pm \sqrt{4}}{2}$$

$$x = \frac{8 \pm 2}{2}$$

$$x = \frac{10}{2} \quad \text{or} \quad x = \frac{6}{2}$$

$$x = 5 \quad \text{or} \quad x = 3$$

Here are some quadratic equations and their solutions. Use the quadratic formula to show that the solutions are correct.

1.  $x^2 + 4x - 5 = 0$ . The solutions are  $x = -5$  and  $x = 1$ .

2.  $x^2 + 7x + 12 = 0$ . The solutions are  $x = -3$  and  $x = -4$ .



3.  $x^2 + 10x + 18 = 0$ . The solutions are  $x = -5 \pm \frac{\sqrt{28}}{2}$ .

4.  $x^2 - 8x + 11 = 0$ . The solutions are  $x = 4 \pm \frac{\sqrt{20}}{2}$ .

5.  $9x^2 - 6x + 1 = 0$ . The solution is  $x = \frac{1}{3}$ .

6.  $6x^2 + 9x - 15 = 0$ . The solutions are  $x = -\frac{5}{2}$  and  $x = 1$ .

 **Are you ready for more?**

1. Use the quadratic formula to solve  $ax^2 + c = 0$ . Let's call the resulting equation P.
2. Solve the equation  $3x^2 - 27 = 0$  in two ways, showing your reasoning for each:
  - Without using any formulas
  - Using equation P
3. Check that you got the same solutions using each method.
4. Use the quadratic formula to solve  $ax^2 + bx = 0$ . Let's call the resulting equation Q.
5. Solve the equation  $2x^2 + 5x = 0$  in two ways, showing your reasoning for each:
  - Without using any formulas
  - Using equation Q
6. Check that you got the same solutions using each method.

## Lesson 16 Summary

We have learned a couple of methods for solving quadratic equations algebraically:

- By rewriting the equation as so that one side is 0 and the other side is in factored form, then using the zero product property
- By completing the square

Some equations can be solved quickly with one of these methods, but many cannot. Here is an example:  $5x^2 - 3x - 1 = 0$ . The expression on the left cannot be rewritten in factored form with rational coefficients. Because the coefficient of the squared term is not a perfect square, and the coefficient of the linear term is an odd number, completing the square would be inconvenient and would result in a perfect square with fractions.

The **quadratic formula** can be used to find the solutions to any quadratic equation, including those that are tricky to solve with other methods.

For an equation of the form  $ax^2 + bx + c = 0$ , where  $a$ ,  $b$ , and  $c$  are numbers and  $a \neq 0$ , the solutions are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation  $5x^2 - 3x - 1 = 0$ , we see that  $a = 5$ ,  $b = -3$ , and  $c = -1$ . Let's solve it!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

the quadratic formula

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(-1)}}{2(5)}$$

Substitute the values of  $a$ ,  $b$ , and  $c$ .

$$x = \frac{3 \pm \sqrt{9 + 20}}{10}$$

Evaluate each part of the expression.

$$x = \frac{3 \pm \sqrt{29}}{10}$$

A calculator gives approximate solutions of 0.84 and -0.24 for  $\frac{3+\sqrt{29}}{10}$  and  $\frac{3-\sqrt{29}}{10}$ .

We can also use the formula for simpler equations like  $x^2 - 9x + 8 = 0$ , but it may not be the most efficient way. If the quadratic expression can be easily rewritten in factored form or made into a perfect square, those methods may be preferable. For example, rewriting  $x^2 - 9x + 8 = 0$  as  $(x - 1)(x - 8) = 0$  immediately tells us that the solutions are 1 and 8.

