

# Completing the Square (Part 2)

Let's solve some harder quadratic equations.

## 13.1

### Math Talk: Equations with Fractions

Solve each equation mentally.

- $x + x = \frac{1}{4}$

- $(\frac{3}{2})^2 = x$

- $\frac{3}{5} + x = \frac{9}{5}$

- $\frac{1}{12} + x = \frac{1}{4}$

## 13.2

### Spot Those Errors!

Here are four equations, followed by worked solutions of the equations. Each solution has at least one error.

- Solve one or more of these equations by completing the square.
- Then, look at the worked solution of the same equation as the one you solved. Find and describe the error or errors in the worked solution.

$$1. \ x^2 + 14x = -24$$

$$2. \ x^2 - 10x + 16 = 0$$

$$3. \ x^2 + 2.4x = -0.8$$

$$4. \ x^2 - \frac{6}{5}x + \frac{1}{5} = 0$$

Worked solutions (with errors):

1.

$$\begin{aligned}x^2 + 14x &= -24 \\x^2 + 14x + 28 &= 4 \\(x + 7)^2 &= 4 \\x + 7 &= 2 \quad \text{or} \quad x + 7 = -2 \\x &= -5 \quad \text{or} \quad x = -9\end{aligned}$$

2.

$$\begin{aligned}x^2 - 10x + 16 &= 0 \\x^2 - 10x + 25 &= 9 \\(x - 5)^2 &= 9 \\x - 5 &= 9 \quad \text{or} \quad x - 5 = -9 \\x &= 14 \quad \text{or} \quad x = -4\end{aligned}$$

3.

$$\begin{aligned}x^2 + 2.4x &= -0.8 \\x^2 + 2.4x + 1.44 &= 0.64 \\(x + 1.2)^2 &= 0.64 \\x + 1.2 &= 0.8 \\x &= -0.4\end{aligned}$$

4.

$$\begin{aligned}x^2 - \frac{6}{5}x + \frac{1}{5} &= 0 \\x^2 - \frac{6}{5}x + \frac{9}{25} &= \frac{9}{25} \\\left(x - \frac{3}{5}\right)^2 &= \frac{9}{25} \\x - \frac{3}{5} &= \frac{3}{5} \quad \text{or} \quad x - \frac{3}{5} = -\frac{3}{5} \\x &= \frac{6}{5} \quad \text{or} \quad x = 0\end{aligned}$$

## 13.3 Solving Some More Quadratic Equations

Solve these equations by completing the square.

$$1. (x - 3)(x + 1) = 5$$

$$2. x^2 + \frac{1}{2}x = \frac{3}{16}$$

$$3. x^2 + 3x + \frac{8}{4} = 0$$

$$4. (7 - x)(3 - x) + 3 = 0$$

$$5. x^2 + 1.6x + 0.63 = 0$$

 **Are you ready for more?**

1. Show that the equation  $x^2 + 10x + 9 = 0$  is equivalent to  $(x + 3)^2 + 4x = 0$ .
2. Write an equation that is equivalent to  $x^2 + 9x + 16 = 0$  and that includes  $(x + 4)^2$ .
3. Does this method help you find solutions to the equations? Explain your reasoning.

## Lesson 13 Summary

Completing the square can be a useful method for solving quadratic equations in cases in which it is not easy to rewrite an expression in factored form. For example, let's solve this equation:

First, we'll add  $\frac{75}{4}$  to each side to make things easier on ourselves.

$$x^2 + 5x - \frac{75}{4} = 0$$

$$x^2 + 5x - \frac{75}{4} + \frac{75}{4} = 0 + \frac{75}{4}$$
$$x^2 + 5x = \frac{75}{4}$$

To complete the square, take  $\frac{1}{2}$  of the coefficient of the linear term, 5, which is  $\frac{5}{2}$ , and square it, which is  $\frac{25}{4}$ . Add this to each side:

$$x^2 + 5x + \frac{25}{4} = \frac{75}{4} + \frac{25}{4}$$
$$x^2 + 5x + \frac{25}{4} = \frac{100}{4}$$

Notice that  $\frac{100}{4}$  is equal to 25, and rewrite it:

$$x^2 + 5x + \frac{25}{4} = 25$$

Since the left side is now a perfect square, let's rewrite it:

$$\left(x + \frac{5}{2}\right)^2 = 25$$

For this equation to be true, one of these equations must true:

$$x + \frac{5}{2} = 5 \quad \text{or} \quad x + \frac{5}{2} = -5$$

To finish up, we can subtract  $\frac{5}{2}$  from each side of the equal sign in each equation.

$$x = 5 - \frac{5}{2} \quad \text{or} \quad x = -5 - \frac{5}{2}$$
$$x = \frac{5}{2} \quad \text{or} \quad x = -\frac{15}{2}$$
$$x = 2\frac{1}{2} \quad \text{or} \quad x = -7\frac{1}{2}$$

It takes some practice to become proficient at completing the square, but it makes it possible to solve many more equations than we could by methods we learned previously.

